

# PUMP SYSTEMS

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# SECTION 8.1

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# GENERAL CHARACTERISTICS OF PUMPING SYSTEMS AND SYSTEM-HEAD CURVES

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J. P. MESSINA

## **SYSTEM CHARACTERISTICS AND PUMP HEAD**

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A pump is used to deliver a specified rate of flow through a particular system. When a pump is to be purchased, this required capacity must be specified along with the total head necessary to overcome resistance flow and to meet the pressure requirements of the system components. The total head rating of a centrifugal pump is usually measured in feet (meters), and the differential pressure rating of a positive displacement pump is usually measured in pounds per square inch (kilopascals or bar<sup>1</sup>). Both express, in equivalent terms, the work in foot-pounds (newton-meters) the pump is capable of doing on each unit weight (force) of liquid pumped at the rated flow.<sup>2</sup> It is the responsibility of the purchaser to determine the required pump total head so the supplier can make a proper pump selection. Underestimating the total head required will result in a centrifugal pump's delivering less than the desired flow through the system. An underestimate of the differential pressure required will result in a positive displacement pump's using more power than estimated, and the design pressure limit of the pump could be exceeded. Therefore, system pressure and resistance to flow, which are dependent on system characteristics, dictate the required pump head rating.

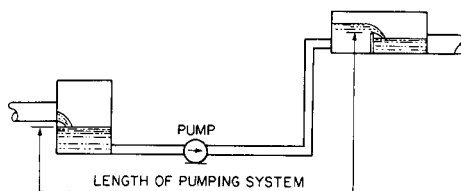
## **THE PUMPING SYSTEM**

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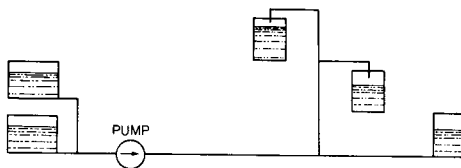
The piping and equipment through which the liquid flows to and from the pump constitute the pumping system. Only the length of the piping containing liquid controlled by the

<sup>1</sup> 1 bar = 1<sup>5</sup> Pa.

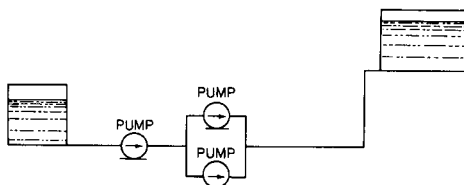
<sup>2</sup>Work per unit weight mass, rather than weight force, is sometimes called *specific delivery work*; it has the units of newton-meters per kilogram and is equal to total head multiplied by *g*, the gravitation constant.



**FIGURE 1** Length of system controlled by pump



**FIGURE 2** Branch-line pumping system



**FIGURE 3** Pumps in series and in parallel

action of the pump is considered part of the system. A pump and the limit of its system length are shown in Figure 1.

The pump suction and discharge piping can consist of branch lines, as shown in Figure 2. There can be more than one pump in a pumping system. Several pumps can be piped together in series or in parallel or both, as shown in Figure 3. When there is more than one pump, the flow through the system is determined by the combined performance of all the pumps.

The system through which the liquid is pumped offers resistance to flow for several reasons. Flow through pipes is impeded by friction. If the liquid discharges to a higher elevation or a higher pressure, additional resistance is encountered. The pump must therefore overcome the total system resistance due to friction and, as required, produce an increase in elevation or pressure at the desired rate of flow. System requirements may be such that the pump discharges to a lower elevation or pressure but additional pump head is still required to overcome pipe friction and obtain the desired rate of flow.

## **ENERGY IN AN INCOMPRESSIBLE LIQUID**

The work done by a pump is the difference between the energy level at the point where the liquid leaves the pump and the energy level at the point where the liquid enters the pump. Work is also the amount of energy added to the liquid in the system. The total energy at any point in a pumping system is a relative term and is measured relative to some arbitrarily selected datum plane.

An incompressible liquid can have energy in the form of velocity, pressure, or elevation. Energy in various forms is added to the liquid as it passes through the pump, and the total energy added is constantly increasing with flow. It is appropriate then to speak of the energy added by a pump as the energy added per unit of weight (force) of the liquid pumped, and the units of energy expressed this way are foot-pounds per pound (newton-meters per newton) or just feet (meters). Therefore, when adding together the energies in their various forms at some point, it is necessary to express each quantity in common equivalent units of feet (meters) of *head*.

Liquid flowing in a conduit can undergo changes in energy form. Bernoulli's theorem for an incompressible liquid states that in steady flow, without losses, the energy at any point in the conduit is the sum of the *velocity head*, *pressure head*, and *elevation head* and that this sum is constant along a streamline in the conduit. Therefore, the energy at any point in the system relative to a selected datum plane is

$$H = \frac{V^2}{2g} + \frac{p}{\gamma} + Z \quad (1)$$

where  $H$  = energy (total head) of system, ft · lb/lb or ft (N · m/N or m)

$V$  = velocity, ft/s (m/s)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

$p$  = pressure, lb/ft<sup>2</sup> (N/m<sup>2</sup>)

$\gamma$  = specific weight (force) of liquid, lb/ft<sup>3</sup> (N/m<sup>3</sup>)

$Z$  = elevation above (+) or below (−) datum plane, ft (m)

The velocity and pressure at the point of energy measurement are expressed in units of *equivalent head* and are added to the distance  $Z$  that this point is above or below the selected datum plane. If pressure is measured as gage (relative to atmosphere), total head  $H$  is gage; if pressure is measured as absolute, total head  $H$  is absolute. Equation 1 can also be applied to liquid at rest in a vertical column or in a large vessel (or to liquids of various densities) to account for changes in pressure with changes in elevation or vice versa.

The equivalent of velocity and pressure energy heads in feet (meters) can be thought of as the height to which a vessel of liquid of constant density has to be filled, above the point of measurement, to create this same velocity or pressure. This is illustrated in Figure 4 and further explained in the following text.

**Velocity Head** The kinetic energy in a mass of flowing liquid is  $\frac{1}{2} mV^2$  or  $\frac{1}{2} (W/g)V^2$ . The kinetic energy per unit weight (force) of this liquid is  $\frac{1}{2} (WV^2/Wg)$ , or  $V^2/2g$ , measured in feet (meters). This quantity is theoretically equal to the equivalent static head of liquid that is required in a vessel above an opening if the discharge is to have a velocity equal to  $V$ . This is also the theoretical height the jet of liquid would rise if it were discharging vertically upward from an orifice.

A free-falling particle in a vacuum acquires the velocity  $V$  starting from rest after falling a distance  $H$ . Also

$$V = \sqrt{2gH}$$

All liquid particles moving with the same velocity have the same velocity head, regardless of specific weight. The velocity of liquid in pipes and open channels invariably varies across any one section of the conduit. However, when calculating system resistance it is sufficiently accurate to use the average velocity, computed by dividing the flow rate by the cross-sectional area of the conduit, when substituting in the term  $V^2/2g$ .

**Pressure Head** The pressure head, or flow work, in a liquid is  $p/\gamma$ , with units in feet (meters). A liquid, having pressure, is capable of doing work, for example, on a piston having an area  $A$  and stroke  $L$ . The quantity of liquid required to complete one stroke is  $\gamma AL$ . The work (force × stroke) per unit weight (force) is  $pAL/\gamma AL$ , or  $p/\gamma$ . The work a pump

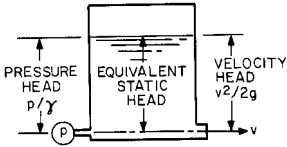
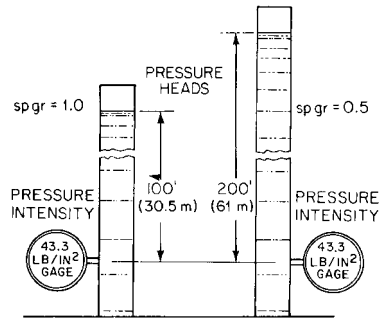


FIGURE 4 Equivalent static head

FIGURE 5 Liquids of different specific weights (also specific gravities) require different column heights or pressure heads to produce the same pressure intensity (43.3 lb/in<sup>2</sup> = 298.5 kPa).

must do to produce pressure intensity in liquids having different specific weights varies inversely with the specific weight or specific gravity of the liquid. Figure 5 illustrates this point for liquids having specific gravities of 1.0 and 0.5. The less dense liquid must be raised to a higher column height to produce the same pressure as the denser liquid. The pressure at the bottom of each liquid column  $H$  is the weight of the liquid above the point of pressure measurement divided by the cross-sectional area  $A$  at the same point,  $AH\gamma/A$ , which is simply  $H\gamma$ . Note that in this discussion  $A$  is in square feet (square meters) and  $L$  and  $H$  are in feet (meters).

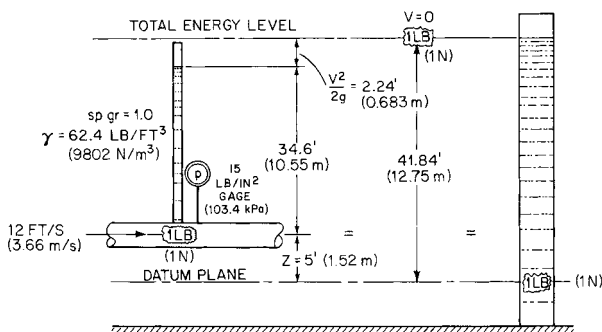
The height of the liquid column in feet (meters) above the point of pressure measurement, if the column is of constant density, is equivalent pressure static head. When pressure intensity  $AH\gamma/A$  is substituted for  $p$  in  $p/\gamma$ , it can be seen that the pressure head  $H$  is the liquid column height. Therefore, at the base of equal columns containing different liquids (with equal surface pressures), the pressure heads in feet (meters) are the same but the intensities in pounds per square foot (newtons per square meter) are different. For this reason, it is necessary to identify the liquids when comparing pressure heads.

**Elevation Head** The elevation energy, or potential energy, in a liquid is the distance  $Z$  in feet (meters) measured vertically above or below an arbitrarily selected horizontal datum plane. Liquid above a reference datum plane has positive potential energy because it can fall a distance  $Z$  and acquire kinetic energy or vertical head equal to  $Z$ . Also, it requires  $WZ$  ft · lb (N · m) of work to raise  $W$  lb (N) of liquid above the datum plane. The work per unit weight (force) of liquid is therefore  $WZ/W$ , or  $Z$  ft (m). In a pumping system, the energy required to raise a liquid above a reference datum plane can be thought of as being provided by a pump located at the datum elevation and producing a pressure that will support the total weight of the liquid in a pipe between the pump discharge and the point in the pipe to which the liquid is to be raised. This pressure is  $AZ\gamma/A$ , or simply  $Z\gamma$  lb/ft<sup>2</sup> (N/m<sup>2</sup>) or  $Z\gamma/144$  lb/in<sup>2</sup>. Because head is equal to pressure divided by specific weight, elevation head is  $Z\gamma/\gamma$ , or  $Z$  ft.

Liquid below the reference datum plane has negative elevation head.

**Total Head** Figure 6 illustrates a liquid under pressure in a pipe. To determine the total head at the pressure gage connection and relative to the datum plane, Eq. 1 may be used (assume that the gage is at the pipe centerline):

$$H = \frac{V^2}{2g} + \frac{p}{\gamma} + Z$$



**FIGURE 6** The total head, or energy, in foot-pounds per pound (newtonmeters per newton) is equal to the sum of the velocity, pressure, and elevation heads relative to a datum place. A unit weight (force) of the same liquid or any liquid raised to rest at the height shown, or under a column of liquid of this same height, has the same head as the unit weight (force) of liquid shown flowing in the pipe.

In USCS units

$$H = \frac{12^2}{2 \times 32.17} + \frac{144 \times 15}{62.4} + 5$$

$$= 2.24 + 34.6 + 5$$

$$= 41.84 \text{ ft} \cdot \text{lb/lb, or ft}$$

In SI units

$$H = \frac{3.66^2}{2 \times 9.807} + \frac{103.4 \times 1000}{9802} + 1.52$$

$$= 0.683 + 10.55 + 1.52$$

$$= 12.75 \text{ N} \cdot \text{m/N, or m}$$

The total head may also be calculated using the expression

$$H = \frac{V^2}{2g} + \text{manometer height} + Z$$

In USCS units

$$H = 2.24 + 34.6 + 5$$

$$= 41.84 \text{ ft} \cdot \text{lb/lb, or ft}$$

In SI units

$$H = 0.683 + 10.55 + 1.52$$

$$= 12.75 \text{ N} \cdot \text{m/N, or m}$$

The total head of 41.84 ft (12.75 m) is equivalent to 1 lb (N) of the liquid raised 41.84 ft (12.75 m) above the datum plane (zero velocity) or the pressure head of 1 lb (N) of the liquid under a column height of 41.84 ft (12.75 m) measured at the datum plane.

The gage pressure  $p$ , and consequently the pressure head, are measured relative to atmospheric pressure. Gage pressure head can therefore be a positive or a negative quantity. The pressure may also be expressed as an absolute pressure (measured from complete vacuum). Therefore, when velocity, pressure, and elevation heads are combined to obtain the total energy at a point, it should be clearly stated that the total head is either feet (meters) gage or feet (meters) absolute with respect to the datum plane.

The pressure or velocity of a liquid may at times be given as a pressure head of a liquid having a density different from the density of the liquid being pumped. In the total head, that is, the sum of the pressure, velocity, and elevation heads, the components must be corrected to be equal to the head of the liquid being pumped. For example, if the pressure is measured by a manometer to be 24 in (61 cm) of mercury (sp. gr. = 13.6) absolute,

the pressure head, or energy, in foot-pounds per pound (newton-meters per newton) of water pumped at 60°F (15.6°C) is found as follows.

Let subscripts 1 and 2 denote different liquids or, in this example, mercury and water, respectively:

$$\begin{aligned} h_1 &= \frac{p_1}{\gamma_1} \\ p_1 &= h_1 \gamma_1 = p_2 \\ h_2 &= \frac{p_2}{\gamma_2} = \frac{h_1 \gamma_1}{\gamma_2} \\ &= \frac{\gamma_1}{\gamma_2} h_1 \end{aligned} \quad (2)$$

$$= \frac{\text{sp. gr.}_1}{\text{sp. gr.}_2} h_1 \quad (3)$$

Therefore

$$\text{in USCS units} \quad h_2 = \frac{13.6}{1} \times \frac{24}{12} = 27.2 \text{ ft abs}$$

$$\text{in SI units} \quad h_2 = \frac{13.6}{1} \times \frac{61}{100} = 8.3 \text{ m abs}$$

Also,  $h_2 = (27.2 - 13.6) \left( \frac{30}{12} \right) = -6.8 \text{ ft}$  [ $(8.3 - 13.6) \left( \frac{76}{100} \right) = -2.04 \text{ m}$ ] gage if corrected to a standard barometer of 30 in (76 cm) of mercury.

## PUMP TOTAL HEAD

The total head of a pump is the difference between the energy level at the pump discharge (point 2) and that at the pump suction (point 1), as shown in Figures 7 and 8. Applying Bernoulli's equation (Eq. 1) at each point, the pump total head  $TH$  in feet (meters) becomes

$$TH = H_d - H_s = \left( \frac{V_d^2}{2g} + \frac{p_d}{\gamma_d} + Z_d \right) - \left( \frac{V_s^2}{2g} + \frac{p_s}{\gamma_s} + Z_s \right) \quad (4)$$

The equation for pump differential pressure  $P_\Delta$  in pounds per square foot (newtons per square meter) is

$$P_\Delta = P_d - P_s = \left[ p_d + \gamma_d \left( Z_d + \frac{V_d^2}{2g} \right) \right] - \left[ p_s + \gamma_s \left( Z_s + \frac{V_s^2}{2g} \right) \right] \quad (5)$$

$$P_\Delta \text{ in lb/in}^2 = P_\Delta \text{ in lb/ft}^2 \div 144$$

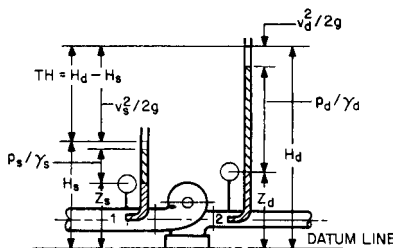


FIGURE 7 Centrifugal pump total head

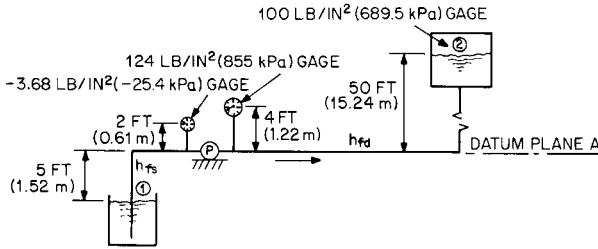


FIGURE 8 Example 1

where the subscripts  $d$  and  $s$  denote discharge and suction, respectively, and

$H$  = total head of system, (+) or (-) ft (m) gage or (+) ft (m) abs

$P$  = total pressure of system, (+) or (-) lb/ft<sup>2</sup> (N/m<sup>2</sup>) gage or (+) lb/ft<sup>2</sup> (N/m<sup>2</sup>) abs

$V$  = velocity, ft/s (m/s)

$p$  = pressure, (+) or (-) lb/ft<sup>2</sup> (N/m<sup>2</sup>) gage or (+) lb/ft<sup>2</sup> (N/m<sup>2</sup>) abs

$Z$  = elevation above (+) or below (-) datum plane, ft (m)

$\gamma$  = specific weight (force) of liquid, lb/ft<sup>3</sup> (N/m<sup>3</sup>)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

Pump total head  $TH$  and pump differential pressure  $P_\Delta$  are always absolute quantities because either gage pressures or absolute pressures but not both are used at the discharge and suction connections of the pump and a common datum plane is selected.

Pump total head in feet (meters) and pump differential pressure in pounds per square foot (newtons per square meter) are related to each other as

$$TH = \frac{P_\Delta}{\gamma} \quad (6)$$

It is very important to note that, if the rated total head of a centrifugal pump is given in *feet (meters)*, this head can be imparted to *all* individual liquids pumped at the rated capacity and speed, regardless of the specific weight (force) of the liquids as long as their viscosities are approximately the same. A pump handling different liquids of approximately the same viscosity will generate the same total head but will not produce the same differential pressure, nor will the power required to drive the pump be the same. On the other hand, a centrifugal pump rated in pressure units would have to have a different pressure rating for each liquid of different specific weight (force). In this section, pump total head will be expressed in feet (meters), the usual way of rating centrifugal pumps. For an explanation of positive displacement pump differential pressure, its use and relationship to pump total head, see Chapter 3.

Pump total head can be measured by installing gages at the pump suction and discharge connections and then substituting these gage readings into Eq. 4. Pump total head may also be found by measuring the energy difference between any two points in the pumping system, one on each side of the pump, providing all losses (other than pump losses) between these points are credited to the pump and added to the energy head difference. Therefore, between any two points in a pumping system where the energy is added only by the pump and the specific weight (force) of the liquid does not change (for example, as a result of temperature), the following general equation for determining pump total head applies:

$$\begin{aligned} TH &= (H_2 - H_1) + \Sigma h_{f(1-2)} \\ &= \left( \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + Z_2 \right) - \left( \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + Z_1 \right) + \Sigma h_{f(1-2)} \end{aligned} \quad (7)$$



where the subscripts 1 and 2 denote points in the pumping system anyplace upstream and downstream from the pump, respectively, and

$H$  = total head of system, (+) or (–) ft (m) gage or (+) ft (m) abs

$V$  = velocity, ft/s (m/s)

$p$  = pressure, (+) or (–) lb/in<sup>2</sup> (N/m<sup>2</sup>) gage or (+) lb/in<sup>2</sup> (N/m<sup>2</sup>) abs

$Z$  = elevation above (+) or below (–) datum plane, ft (m)

$\gamma$  = specific weight (force) of liquid (assumed the same between points), lb/ft<sup>3</sup> (N/m<sup>3</sup>)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

$\Sigma h_f$  = sum of piping losses between points, ft (m)

When the specific gravity of the liquid is known, the pressure head may be calculated from the following relationships:

$$\text{In feet} \quad \frac{p}{\gamma} = \frac{0.016 \text{ lb/ft}^2}{\text{sp. gr.}} \text{ or } \frac{2.3 \text{ lb/in}^2}{\text{sp. gr.}} \quad (8a)$$

$$\text{In meters} \quad \frac{p}{\gamma} = \frac{0.102 \text{ kPa}}{\text{sp. gr.}} \text{ or } \frac{1.02 \times 10^{-3} \text{ bar}}{\text{sp. gr.}} \quad (8b)$$

The velocity in a pipe may be calculated as follows:

$$\text{In feet per second} \quad V = \frac{(\text{gm})(0.408)}{(\text{pipe ID in inches})^2} \quad (9a)$$

$$\text{In meters per second} \quad V = \frac{(\text{m}^3/\text{h})(3.54)}{(\text{pipe ID in cm})^2} \text{ or } \frac{(\text{liters/s})(12.7)}{(\text{pipe ID in cm})^2} \quad (9b)$$

The following example illustrates the use of Eqs. 4 and 7 for determining pump total head.

**EXAMPLE 1** A centrifugal pump delivers 1000 gpm (227 m<sup>3</sup>/hr) of liquid of specific gravity 0.8 from the suction tank to the discharge tank through the piping shown in Figure 8. (a) Calculate pump total head using gages and the datum plane selected. (b) Calculate total head using the pressures at points 1 and 2 and the same datum plane as (a).

*Given:* Suction pipe ID = 8 in (203 mm), discharge pipe ID = 6 in (152 mm),  $h_f$  = pipe, valve, and fitting losses,  $h_{fs}$  = 3 ft (0.91 mm),  $h_{fd}$  = 25 ft (7.62 m).

$$\text{In USCS units} \quad \text{Calculated pipe velocity} = \frac{(\text{gpm})(0.408)}{(\text{ID in inches})^2}$$

$$V_s = \frac{1000 \times 0.408}{8^2} = 6.38 \text{ ft/s}$$

$$V_d = \frac{1000 \times 0.408}{6^2} = 11.33 \text{ ft/s}$$

$$\text{In SI units} \quad \text{Calculated pipe velocity} = \frac{(\text{m}^3/\text{h})(3.54)}{(\text{ID in cm})^2}$$

$$V_s = \frac{227 \times 3.54}{20.3^2} = 1.95 \text{ m/s}$$

$$V_d = \frac{227 \times 3.54}{15.2^2} = 3.48 \text{ m/s}$$

(a) From Eq. 4,

$$TH = \left( \frac{V_d^2}{2g} + \frac{p_d}{\gamma_d} + Z_d \right) - \left( \frac{V_s^2}{2g} + \frac{p_s}{\gamma_s} + Z_s \right)$$

and Eq. 8,

$$\text{in USCS units} \quad \frac{p}{\gamma} = \frac{2.31 \text{ lb/in}^2}{\text{sp. gr.}}$$

$$\text{in SI units} \quad \frac{p}{\gamma} = \frac{0.102 \text{ kPa}}{\text{sp. gr.}}$$

Therefore,

in USCS units

$$\begin{aligned} TH &= \left( \frac{11.33^2}{2 \times 32.3} + \frac{2.31 \times 124}{0.8} + 4 \right) - \left( \frac{638^2}{2 \times 32.2} + \frac{2.31(-3.68)}{0.8} + 2 \right) \\ &= 364 - (-8) = 372 \text{ ft} \cdot \text{lb/lb, or ft} \end{aligned}$$

In SI units

$$\begin{aligned} TH &= \left( \frac{3.48^2}{2 \times 9.807} + \frac{0.102 \times 855}{0.8} + 1.22 \right) - \left( \frac{195^2}{2 \times 9.807} + \frac{0.102(-25.4)}{0.8} + 2 \right) \\ &= 110.9 - (-2.43) = 113.3 \text{ N} \cdot \text{m/N, or m} \end{aligned}$$

(b) from Eq. 7,

$$TH = \left( \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + Z_2 \right) - \left( \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + Z_1 \right) + \Sigma h_{f(1-2)}$$

and Eq. 8,

$$\text{in USCS units} \quad \frac{p}{\gamma} = \frac{2.31 \text{ lb/in}^2}{\text{sp. gr.}}$$

$$\text{in SI units} \quad \frac{p}{\gamma} = \frac{0.102 \text{ kPa}}{\text{sp. gr.}}$$

Therefore

$$\begin{aligned} \text{in USCS units} \quad TH &= \left( 0 + \frac{2.31 \times 100}{0.8} + 50 \right) - (0 + 0 - 5) + (3 + 25) \\ &= 399 - (-5) + 28 = 372 \text{ ft} \cdot \text{lb/lb, or ft} \end{aligned}$$

in SI units

$$TH = \left( 0 + \frac{0.102 \times 689.5}{0.8} + 15.24 \right) - (0 + 0 - 1.52) + (0.91 + 7.62)$$

$$= 103.2 - (-1.52) + 8.53 = 113.3 \text{ N} \cdot \text{m/N, or m}$$

## ENERGY AND HYDRAULIC GRADIENT

The total energy at any point in a pumping system may be calculated for a particular rate of flow using Bernoulli's equation (Eq. 1). If some convenient datum plane is selected and the total energy, or head, at various locations along the system is plotted to scale, the line drawn through these points is called the *energy gradient*. Figure 9 shows the variation in total energy  $H$  measured in feet (meters) from the suction liquid surface point 3 to the discharge liquid surface point 4. A horizontal energy gradient indicates no loss of head.

The line drawn through the sum of the pressure and elevation heads at various points represents the pressure variation in flow measured above the datum plane. It also represents the height the liquid would rise in vertical columns relative to the datum plane when the columns are placed at various locations along pipes having positive pressure anywhere in the system. This line, shown dotted in Figure 9, is called the *hydraulic gradient*. The difference between the energy gradient line and the hydraulic gradient line is the velocity head in the pipe at that point.

The pump total head is the algebraic difference between the total energy at the pump discharge (point 2) and the total energy at the pump suction (point 1).

## SYSTEM-HEAD CURVES

A pumping system may consist of piping, valves, fittings, open channels, vessels, nozzles, weirs, meters, process equipment, and other liquid-handling conduits through which flow is required for various reasons. When a particular system is being analyzed for the pur-

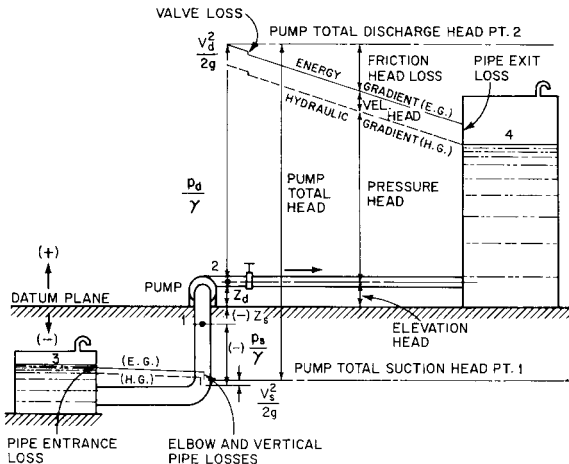


FIGURE 9 Energy and hydraulic gradients

pose of selecting a pump or pumps, the resistance to flow of the liquid through these various components must be calculated. It will be explained in more detail later in this section that the resistance increases with flow at a rate approximately equal to the square of the flow through the system. In addition to overcoming flow resistance, it may be necessary to add head to raise the liquid from suction level to a higher discharge level. In some systems the pressure at the discharge liquid surface may be higher than the pressure at the suction liquid surface, a condition that requires more pumping head. The latter two heads are *fixed system heads*, as they do not vary with rate of flow. Fixed system heads can also be negative, as would be the case if the discharge level elevation or the pressure above that level were lower than suction elevation or pressure. Fixed system heads are also called *static heads*.

A system-head curve is a plot of total system resistance, variable plus fixed, for various flow rates. It has many uses in centrifugal pump applications. It is preferable to express system head in feet (meters) rather than in pressure units because centrifugal pumps are rated in feet (meters), as previously explained. System-head curves usually show flow in gallons per minute, but when large quantities are involved, the units of cubic feet per second or million gallons per day are used. Although the standard SI units for volumetric flow are cubic meters per second, the units of cubic meters per hour are more common.

When the system head is required for several flows or when the pump flow is to be determined, a system-head curve is constructed using the following procedure. Define the pumping system and its length. Calculate (or measure) the fixed system head, which is the net change in total energy from the beginning to the end of the system due to elevation or pressure head differences. An increase in head in the direction of flow is a positive quantity. Next, calculate, for several flow rates, the variable system total head loss through all piping, valves fittings, and equipment in the system. As an example, see Figure 10, in which the pumping system is defined as starting at point 1 and ending at point 2. The fixed system head is the net change in total energy. The total head at point 1 is  $p_s/\gamma$ , and that at point 2 is  $p_d/\gamma + Z$ . The pressure and liquid levels do not vary with flow. The variable system head is pipe friction (including valves and fittings). The fixed head and variable heads for several flow rates are added together, resulting in a curve of total system head versus flow.

The flow produced by a centrifugal pump varies with the system head, whereas the flow of a positive displacement pump is independent of the system head. By superimposing the head-capacity characteristic curve of a centrifugal pump on a system-head curve, as shown in Figure 10, the flow of a pump can be determined. The curves will intersect at the flow rate of the pump, as this is the point at which the pump head is equal to the required system

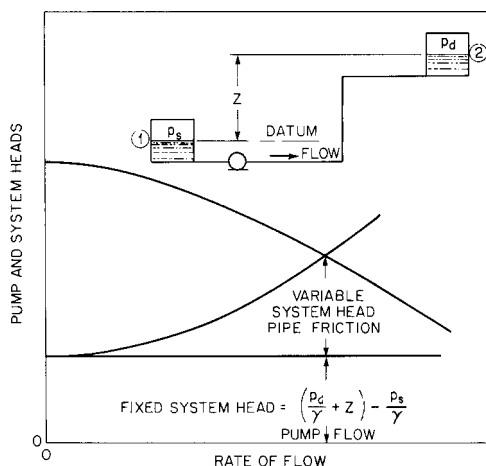


FIGURE 10 Construction of system total-head curve

head for the same flow. When a pump is being purchased, it should be specified that the pump head-capacity curve intersect the system-head curve at the desired flow rate. This intersection should be at the pump's best efficiency capacity or very close to it.

The system-head curve for Example 1 is shown in Figure 11. This assumes that the suction and discharge liquid levels are 5 ft (1.5 m) below and 50 ft (15 m) above the datum plane, respectively, and do not vary with flow. The pressure in the discharge tank is also independent of flow and is 100 lb/in<sup>2</sup> (689.5 kPa) gage. These values are therefore fixed system heads. The pipe and fitting losses are assumed to vary with flow as a square function. The length of the pumping system is from point 1 to point 2. The difference in heads at these points plus the frictional losses at various flow rates are the total system head and the head required by a pump for the different flows. It is necessary to calculate the total system head for only one flow rate—say, design—which in this example is 1000 gpm (227 m<sup>3</sup>/h). The total head at other flow conditions is the fixed system head plus the variable system head multiplied by  $(\text{gpm}/1000)^2 (\text{m}^3/\text{h} \div 227)^2$ . If Example 1 is an existing system, the total head may be calculated by using gages at the pump suction and discharge connections. The total head measured will then be the head at the intersection of the pump and system curves, as shown in Figure 11. In this example, a correctly purchased pump would produce a total head of 372 ft (113 m) at the design flow of 1000 gpm (227 m<sup>3</sup>/h).

In systems that are open-ended and in which there is a decrease in elevation from inlet to outlet, a portion of the system-head curve will be negative (Figure 12). In this example, the pump is used to increase gravity flow. Without a pump in the system, the negative resistance, or static head, is the driving head that moves the liquid through the system. Steady-state gravity flow is sustained at the flow rate corresponding to zero total system head (negative static head plus system resistance equals zero). If a flow is required at any rate greater than that which gravity can produce, a pump is required to overcome the additional system resistance.

For additional information concerning the construction of system-head curves for flow in branch lines, refer to Section 8.2.

## VARIANTS IN PUMPING SYSTEMS

For a fixed set of conditions in a pumping system, there is just one total head for each flow rate. Consequently, a centrifugal pump operating at a constant speed can deliver just one flow. In practice, however, conditions in a system vary as a result of either controllable or uncontrollable changes. Changes in the valve opening in the pump discharge or bypass line, changes in the suction or discharge liquid level, changes in the pressures at these levels, the aging of pipes, changes in the process, changes in the number of pumps pumping

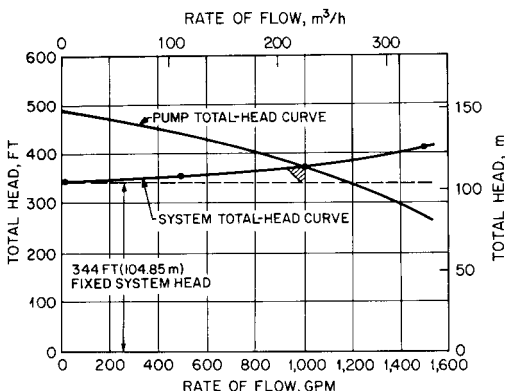


FIGURE 11 System total-head curve for Example 1

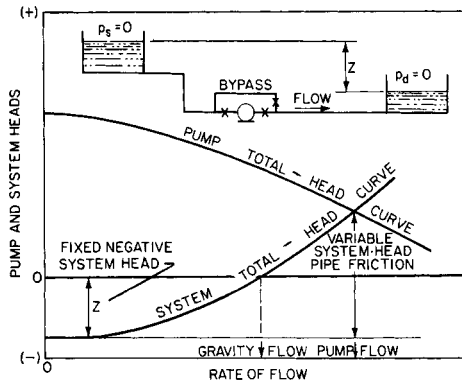


FIGURE 12 Construction of system total-head curve to determine gravity flow and centrifugal pump flow

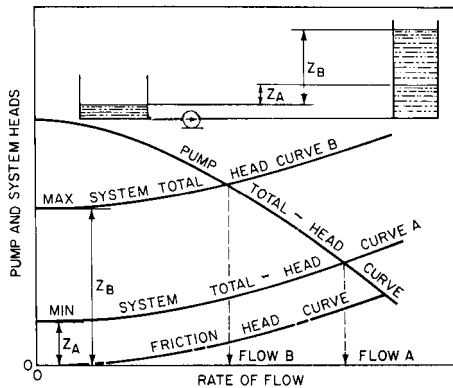


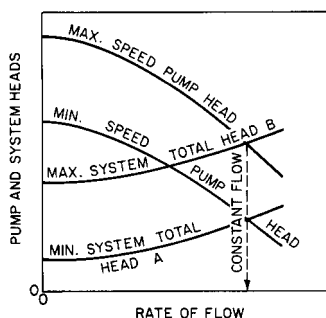
FIGURE 13 Construction of system total-head curves for a pumping system having variable static head

into a common header, changes in the size, length, or number of pipes are all examples of either controllable or uncontrollable system changes. These changes in system conditions alter the shape of the system-head curve and, in turn, affect pump flow.

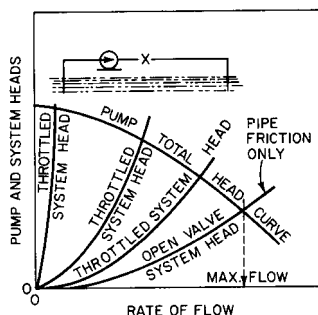
Methods of constructing system-head curves and determining the resultant pump flows for two of the more common of these variants are explained here.

**Variable Static Head** In a system where a pump is taking suction from one reservoir and filling another, the capacity of a centrifugal pump will decrease with an increase in static head. The system-head curve is constructed by plotting the variable system friction head versus flow for the piping. To this is added the anticipated minimum and maximum static heads (difference in discharge and suction levels). The resulting two curves are the total system heads for each condition. The flow rate of the pump is the point of intersection of the pump head-capacity curve with either one of the latter two system-head curves or with any intermediate system-head curve for other level conditions. A typical head versus flow curve for a varying static head system is shown in Figure 13.

If it is desired to maintain a constant pump flow for different static head conditions, the pump speed can be varied to adjust for an increase or decrease in the total system head. A typical variable-speed centrifugal pump operating in a varying static head system can have a constant flow, as shown in Figure 14.



**FIGURE 14** Varying centrifugal pump speed to maintain constant flow for the different reservoir levels shown in Figure 13



**FIGURE 15** Construction of system total-head curves for various valve openings

It is important to select a pump that will have its best efficiency within the operating range of the system and preferably at the condition at which the pump will operate most often.

**Variable System Resistance** A valve or valves in the discharge line of a centrifugal pump alter the variable frictional head portion of the total system-head curve and consequently the pump flow. Figure 15, for example, illustrates the use of a discharge valve to change the system head for the purpose of varying pump flow during a shop performance test. The maximum flow is obtained with a completely open valve, and the only resistance to flow is the friction in the piping, fittings, and flowmeter. A closed valve results in the pump's operating at shutoff conditions and produces maximum head. Any flow between maximum and shutoff can be obtained by proper adjustment of the valve opening.

### ***DIVIDING TOTAL HEAD AND SYSTEM-HEAD CURVES FOR CENTRIFUGAL PUMPS IN SERIES***

Pump limitations or system component requirements may determine that two or more pumps must be used in series. There are practical limitations as to the maximum head that can be developed in a single pump, even if multistaged. When pumping through several system components, there may be pressure limitations that prevent using a single pump to develop all of the head required at the beginning of the system. If several pumps are to be used in series, how should the total head be divided among them?

The sum of the total heads of the pumps must be equal to the required total system head at the design flow. Although mathematically any division of the total head among the pumps to be used is possible (at long as the sum of the pump heads is equal to the total system head), the actual pressure required at various locations along the system flow path determines how the pumping heads are to be divided. An energy or pressure gradient should be drawn for the system. The number of pumps, their locations, and their total heads should be selected to produce the desired pressures (or range of pressures) at critical locations along the system. In addition to considering the pressure loss through components to overcome resistance to flow, consideration should also be given to the minimum pressures required to prevent flashing in piping, cavitation at pump inlets, and so on, as well as the maximum working pressures for different parts of the system.

If preferred, the total system can be divided into subsystems, one for each pump (or group of pumps). The end of one subsystem and the beginning of another can be selected anywhere between pumps in series because the pump total head will be unaffected by the division line. Consequently, several system-head curves can be drawn for specification and

purchasing purposes—for example, primary condensate pump system, secondary condensate pump system, feed pump system, in a total power plant system.

## TRANSIENTS IN SYSTEM HEADS

During the starting of a centrifugal pump and prior to the time normal flow is reached, certain transient conditions can produce or require heads and consequently torques much higher than design. In some cases, the selection of the driver and the pump must be based on starting rather than normal flow conditions.

Low- and medium-specific-speed pumps of the radial- and mixed-flow types (less than approximately 5000 specific speed, rpm, gpm, ft units) have favorable starting characteristics. The pump head at shutoff is not significantly higher than that at normal flow, and the shutoff torque is less than that at normal flow. High-specific-speed pumps of the mixed- and axial-flow types (greater than approximately 5000 specific speed) develop relatively high shutoff heads, and their shutoff torque is greater than that at normal flow. These characteristics of high-specific-speed pumps require special attention during the starting period. Characteristics of pumps of different specific speeds are shown in Figures 16a and 16b.

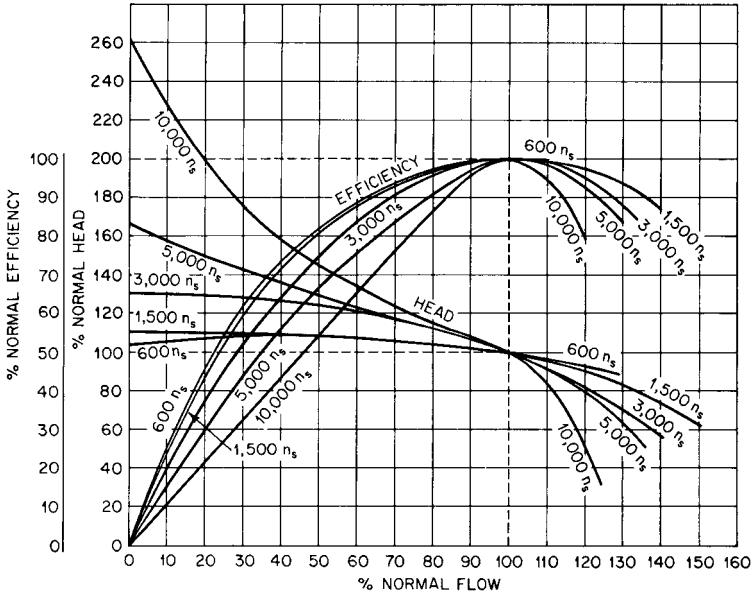
**Starting Against a Closed Valve** When any centrifugal pump is started against a closed discharge valve, the pump head will be higher than normal. The shutoff head will vary with pump specific speed. As shown in Figure 16a, the higher the specific speed, the higher the shutoff head in percent of normal pump head. As a pump is accelerated from rest to full speed against a closed valve, the head on the pump at any speed is equal to the square of the ratio of the speed to the full speed times the shutoff head at full speed. Therefore, during starting, the head will vary from point *A* to point *E* in Figure 17. Points *B*, *C*, and *D* represent intermediate heads at intermediate speeds. The pump, the discharge valve, and any intermediate piping must be designed for maximum head at point *E*.

Pumps requiring less shutoff power and torque than at normal flow condition are usually started against a closed discharge valve. To prevent backflow from a static discharge head prior to starting, either a discharge shutoff valve, a check valve, or a broken siphon is required. When pumps are operated in parallel and are connected to a common discharge header that would permit flow from an operating pump to circulate back through an idle pump, a discharge valve or check valve must be used.

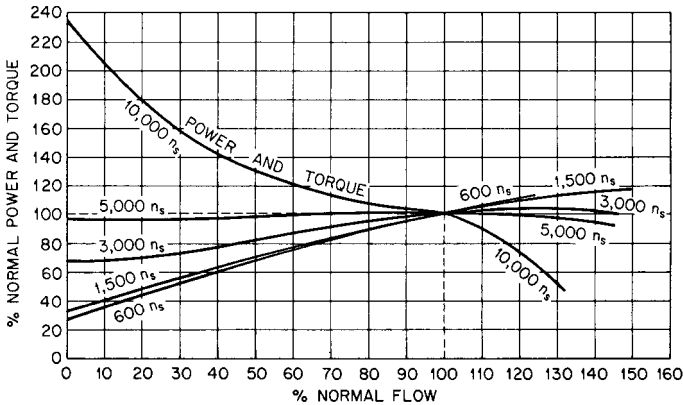
Figure 18 is a typical characteristic curve for a low-specific-speed pump. Figure 19 illustrates the variation of torque with pump speed when the pump is started against a closed discharge valve. The torque under shutoff conditions varies as the square of the ratio of speeds, similar to the variation in shutoff head, and is shown as curve *ABC*. At zero speed, the pump torque is not zero as a result of static friction in the pump bearings and stuffing box or boxes. This static friction is greater than the sum of running friction and power input to the impeller at very low speeds, which explains the dip in the pump torque curve between 0 and 10% speed. Also shown in Figure 19 is the speed-torque curve of a typical squirrel-cage induction motor. Note that the difference between motor and pump torque is the excess torque available to accelerate the pump from rest to full speed. During acceleration, the pump shaft must transmit not only the pump torque (curve *ABC*) but also the excess torque available in the motor. Therefore pump shaft torque follows the motor speed-torque curve less the torque required to accelerate the mass inertia ( $WK^2$ ) of the motor's rotor.

High-specific-speed pumps, especially propeller pumps, requiring more than normal torque at shutoff are not normally started with a closed discharge valve because larger and more expensive drivers would be required. These pumps will also produce relatively high pressures in the pump and in the system between pump and discharge valve. Figure 20 is a typical characteristic curve for a high-specific-speed pump. Curve *ABC* of Figure 21 illustrates the variation of torque with speed when this pump is started against a closed discharge valve. A typical speed-torque curve of a squirrel-cage induction motor sized for normal pump torque is also shown. Note that the motor has insufficient torque to accelerate to full speed and would remain overloaded at point *C* until the discharge valve on the



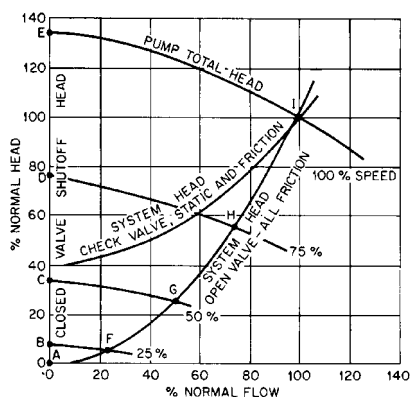


**FIGURE 16A** Approximate comparison of head and efficiency versus flow for impellers of different specific speeds in single-stage volute pumps. Specific speed  $n_s = \text{rpm} \sqrt{\text{gpm}/\text{TH}}$  in  $\text{ft}^{3/4}$   
To convert to other units using  
rpm,  $\text{m}^3/\text{s}$ , m: multiply by 0.01936; rpm,  $\text{m}^3/\text{h}$ , m: multiply by 1.163; rpm, L/s, m: multiply by 0.6123  
To convert to the universal specific speed  $\Omega_s$  (defined in Section 2.1) divide  $n_s$  by 2733.

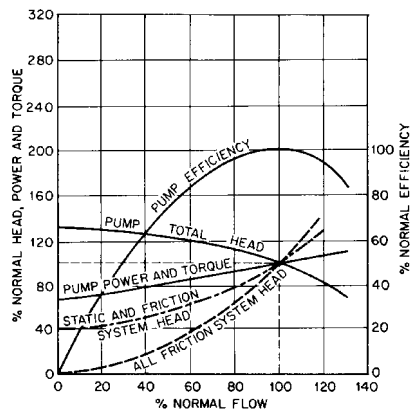


**FIGURE 16B** Approximate comparison of power and torque versus flow for impellers of different specific speeds in single-stage volute pumps

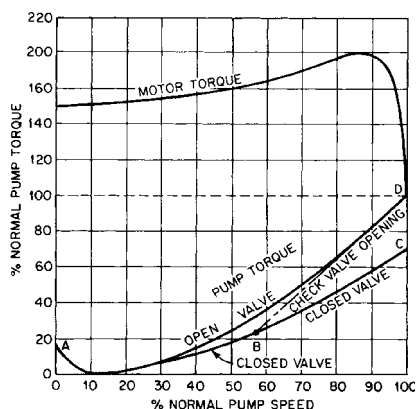
pump was opened. To avoid this situation, the discharge valve should be timed to open sufficiently to keep the motor from overloading when the pump reaches full speed. To accomplish this timing, it may be necessary to start opening the valve in advance of energizing the motor. Care should be taken not to start opening the discharge valve too soon because



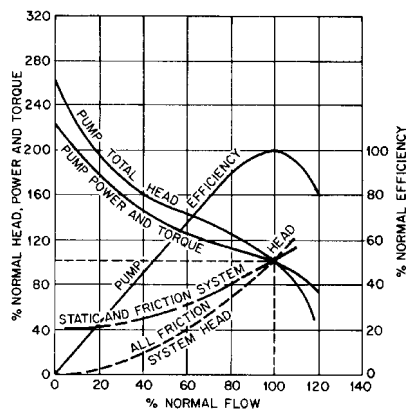
**FIGURE 17** Variation in head when a centrifugal pump is started with a closed valve, an open valve, and a check valve



**FIGURE 18** Typical constant-speed characteristic curves for a low-specific-speed pump



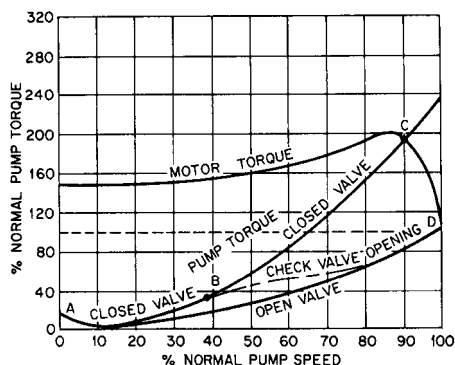
**FIGURE 19** Variation of torque during start-up of a low-specific-speed pump with a closed valve, an open valve, and a check valve. See Figure 18 for pump characteristics.



**FIGURE 20** Typical constant-speed characteristic curves for a high-specific-speed pump

this would cause excessive reverse flow through the pump and require the motor to start under adverse reverse-speed conditions. If the driver is a synchronous motor, additional torque at pull-in speed is required, over and above that needed to overcome system head and accelerate the pump and driver rotors from rest. At the critical pull-in point, sufficient torque must be available to pull the load into synchronism in the prescribed time. A synchronous motor is started on low-torque squirrel-cage windings prior to excitation of the field windings at the pull-in speed. The low starting torque, the torque required at pull-in, and possible voltage drop, which will lower the motor torque (varies as the square of the voltage), must all be taken into consideration when selecting a synchronous motor to start a high-specific-speed pump against a closed valve.

If a high-specific-speed pump is to be started against a closed discharge valve, high starting torques can also be avoided by the use of a bypass valve (see Section 8.2) or by an adjustable-blade pump.<sup>1</sup>



**FIGURE 21** Variation of torque during start-up of a high-specific-speed pump with a closed valve, an open valve, and a check valve. See Figure 20 for pump characteristics.

**Starting Against a Check Valve** A check valve can be used to prevent reverse flow from static head or head from other pumps in the system. The check valve will open automatically when the head from the pump exceeds system head. When a centrifugal pump is started against a check valve, pump head and torque follow shutoff values until a speed is reached at which shutoff head exceeds system head. As the valve opens, the pump head continues to increase, and, at any flow, the head will be that necessary to overcome system static head or head from other pumps, frictional head, valve head loss, and the inertia of the liquid being pumped.

Figure 19, curve *ABD*, illustrates speed-torque variation when a low-specific-speed pump is started against a check valve with static head and system friction as shown in Figure 18. Figure 21, curve *ABD*, illustrates speed-torque variation for a high-specific-speed pump started against a cheek valve with static head and system friction as shown in Figure 20. The use of a quick-opening check valve with high-specific-speed pumps eliminates starting against higher than full-open valve shutoff heads and torques.

The speed-torque curves shown for the period during the acceleration of the liquid in the system have been drawn with the assumption that the head required to accelerate the liquid and overcome inertia is insignificant. Acceleration head is discussed in more detail later.

**Starting Against an Open Valve** If a centrifugal pump is to take suction from a reservoir and discharge to another reservoir having the same liquid elevation or the same equivalent total pressure, it can be started without a shutoff discharge valve or check valve. The system-head curve is essentially all frictional plus the head required to accelerate the liquid in the system during the starting period. Neglecting liquid inertia, the pump head would not be greater than normal at any speed during the starting period, as shown in curve *AFGHI* of Figure 17. Pump torque would not be greater than normal at any speed during the starting period, as shown in Figures 19 and 21, curves *AD*. Pump head and torque at any speed are equal to their values at normal condition times the square of the ratio of the speed to full speed, whereas the capacity varies directly with this ratio.

**Starting a Pump Running in Reverse** When a centrifugal pump discharges against a static head or into a common discharge header with other pumps and is then stopped, the flow will reverse through the pump unless the discharge valve is closed or unless there is a check valve in the system or a broken siphon in a siphon system. If the pump does not have a nonreversing device, it will turn in the reverse direction. A pump that discharges against a static head through a siphon system without a valve will have reverse flow and speed when the siphon is being primed prior to starting.

Figures 22 and 23 illustrate typical reverse-speed-torque characteristics for a low- and a high-specific-speed pump. When flow reverses through a pump and the driver offers very

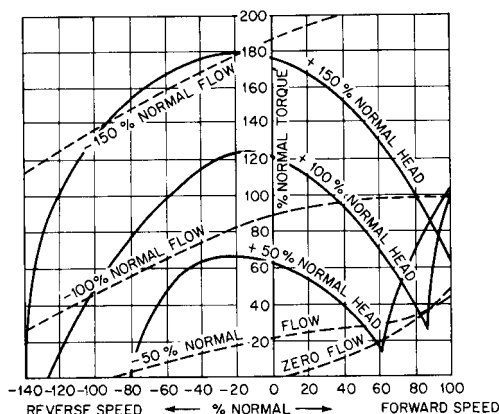


FIGURE 22 Typical reverse-speed-torque characteristics of a low-specific-speed, radial-flow, double-suction pump

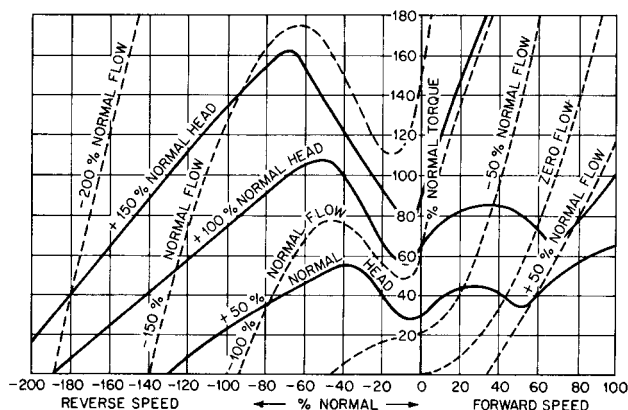


FIGURE 23 Typical reverse-speed-torque characteristics of a high-specific-speed, axial-flow, diffuser pump

little or no torque resistance, the pump will reach higher than normal forward speed in the reverse direction. This runaway speed will increase with specific speed and system head. Shown in Figures 22 and 23 are speed, torque, head, and flow, all expressed as a percentage of the pump design conditions for the normal forward speed. When a pump is running in reverse as a turbine under no load, the head on the pump will be static head (or head from other pumps) minus head loss as a result of friction due to the reverse flow.

If an attempt is made to start the pump while it is running in reverse, an electric motor must apply positive torque to the pump while the motor is initially running in a negative direction. Figures 22 and 23 show, for the two pumps, the torques required to decelerate, momentarily stop, and then accelerate the pump to normal speed. If either of these pumps were pumping into an all-static-head system, starting it in reverse would require overcoming 100% normal head, and it can be seen that a torque in excess of normal would be required by the driver while the driver is running in reverse. In addition to overcoming positive head, the driver must add additional torque to the pump to change the direction of the liquid. This could result in a prolonged starting time under higher than normal current demand. Characteristics of the motor, pump, and system must be analyzed together



where  $L$  = length of constant-cross-section conduit, ft (m)

$\Delta V$  = velocity change, ft/s (m/s)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

$\Delta t$  = time interval, s

To calculate the time to accelerate a centrifugal pump from rest or from some initial speed to a final speed, and to estimate the pump head variation during this interim, a trial-and-error solution may be used. Divide the speed change into several increments of equal no-flow heads, such as *OGHIKL* in Figure 24. For the first incremental speed change, point *O* to point *N*<sub>1</sub>, estimate the total system head, point *M*, between *G* and *A*. Next estimate the total system head, point *X*, at the average speed for this first incremental speed change. These points are shown in Figure 24. Calculate the time in seconds for this incremental speed change to take place using the equation

$$\text{in USCS units} \quad \Delta t = \frac{\Sigma WK^2 \Delta N}{307(T_D - T_P)} \quad (11a)$$

where  $\Sigma WK^2$  = total pump and motor rotor weight moment of inertia,  $W$  = weight (force),  $K$  = radius of gyration, lb · ft<sup>2</sup>

$\Delta N$  = incremental speed change, rpm

$T_D$  = motor torque at average speed, ft · lb

$T_P$  = pump torque at average speed, ft · lb

$$\text{in SI units} \quad \Delta t = \frac{\Sigma MK^2 \Delta N}{9.55(T_D - T_P)} \quad (11b)$$

where  $\Sigma MK^2$  = total pump and motor rotor mass moment of inertia,  $M$  = weight (mass),  $K$  = radius of gyration, kg · m<sup>2</sup>

$\Delta N$  = incremental speed change, rpm

$T_D$  = motor torque at average speed, N · m

$T_P$  = pump torque at average speed, N · m

In SI units, when diameter of gyration  $D$  is used rather than radius of gyration  $K$ , and  $MD^2 = 4MK^2$ , then

$$\Delta t = \frac{\Sigma MD^2 \Delta N}{38.2(T_D - T_P)} \quad (11c)$$

Calculate the acceleration head required to change the flow in the system from point *O* to point *M* using Eq. 10 and time from Eq. 11. Add acceleration head to frictional head at the assumed average flow, and if this value is correct, it will fall on the average pump head-capacity curve, point *X*. Adjust points *M* and *X* until these assumed flows result in the total acceleration and frictional heads agreeing with flow *X* at the average speed. Repeat this procedure for other increments of speed change, adding incremental times to get total accelerating time to bring the pump up to its final speed. Plot system head versus average flow for each incremental speed change during this transient period, as shown in Figure 24.

Figures 25 and 26 illustrate how driver and pump torques can be determined from their respective speed-torque curves. Figure 25 is a family of curves that represents the torques required to produce flow against different heads without acceleration of the liquid or pump for the various speeds selected. Pump torque for any reduced speed can be calculated from the full-speed curve using the relation that torque varies as the second power and flow varies as the first power of the speed ratio. Point *X* is the torque at the average speed and the trial average flow during the first incremental speed change,



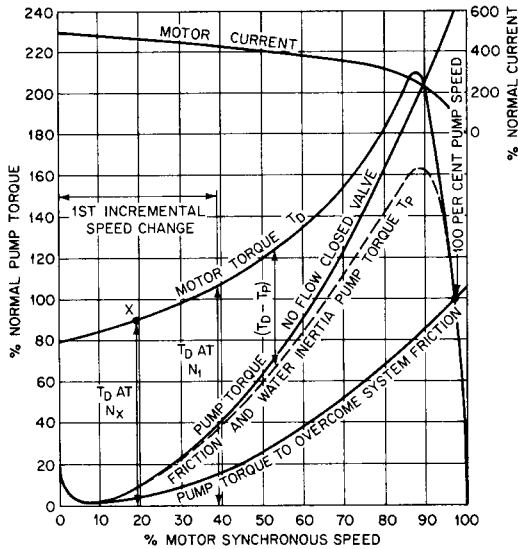


FIGURE 26 Propeller pump and motor speed-torque curves, showing effect of accelerating the liquid in the system during the starting period

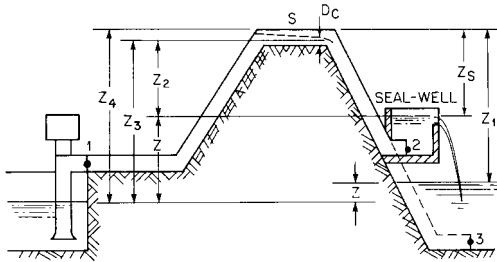


FIGURE 27 Pumping system using a siphon for head recovery

**Siphon Head** Between any two points having the same elevation in a pumping system, no head is lost because of piping elevation changes because the net change in elevation is zero. If the net change in elevation between two points is not zero, additional pump head is required if there is an increase in elevation and less head is required if there is a decrease in elevation.

When piping is laid over and under obstacles with no net change in elevation, no pumping head is required to sustain flow other than that needed to overcome frictional and minor losses. As the piping rises, the liquid pressure head is transformed to elevation head, and the reverse takes place as the piping falls. A pipe or other closed conduit that rises and falls is called a *siphon*, and one that falls and rises is called an *inverted siphon*. The siphon principle is valid provided the conduit flows full and free of liquid vapor and air so the densities of the liquid columns are alike. It is this requirement that determines the limiting height of a siphon for complete recovery because the liquid can vaporize under certain conditions.

Pressure in a siphon is minimum at the summit, or just downstream from it, and Bernoulli's equation can be used to determine if the liquid pressure is above or below vapor pressure. Referring to Figure 27, observe the following. The absolute pressure head  $H_s$  in feet (meters) at the top of the siphon is



$$H_S = H_B - Z_S + h_{f(S-2)} - \frac{V_S^2}{2g} \quad (12)$$

where  $H_B$  = barometric pressure head of liquid pumped, ft (m)

$Z_S$  = siphon height to top of conduit (=  $Z_1$  if no seal well is used), ft (m)

$h_{f(S-2)}$  = frictional and minor losses from  $S$  to 2 (or 3 if no seal well is used), including exit velocity head loss at 2 (or 3), ft (m)

$V_S^2/2g$  = velocity head at summit, ft (m)

The absolute pressure head at the summit can also be calculated using conditions in the up leg by adding the barometric pressure head to the pump head ( $TH$ ) and deducting the distance from suction level to the top of the conduit ( $Z_4$ ), the frictional loss in the up leg ( $h_{f(1-S)}$ ), and the velocity head at the summit. If the suction level is higher than the discharge level and flow is by gravity, the absolute pressure head at the summit is found as above and  $TH = 0$ .

Whenever  $Z_1$  in Figure 27 is so high that it exceeds the maximum siphon capability, a seal well is necessary to increase the pressure at the top of the siphon above vapor pressure. Note  $Z_1 - Z_S$  represents an unrecoverable head and increases the pumping head. Water has a vapor pressure of 0.77 ft (23.5 cm) at 68°F (20°C) and theoretically a 33.23-ft-high (10.13-m) siphon is possible with a 34-ft (10.36-m) water barometer. In practice, higher water temperatures and lower barometric pressures limit the height of siphons used in condenser cooling water systems to 26 to 28 ft (8 to 8.5 m). The siphon height can be found by using Eq. 12 and letting  $H_S$  equal the vapor pressure in feet (meters).

In addition to recovering head in systems such as condenser cooling water, thermal dilution, and levees, siphons are also used to prevent reverse flow after pumping is stopped by use of an automatic vacuum breaker located in the summit. Often siphons are used solely to eliminate the need for valves or flap gates.

In open-ended pumping systems, siphons can be primed by external means of air removal. Unless the siphon is primed initially upon starting, a pump must fill the system and provide a minimum flow to induce siphon action. During this filling period and until the siphon is primed, the siphon head curve must include this additional siphon filling head, which must be provided by the pump. Pumps in siphon systems are usually low-head, and they may not be capable of filling the system to the top of the siphon or of filling it with adequate flow. Low-head pumps are high-specific-speed and require more power at reduced flows than during normal pumping. Figure 28 illustrates the performance of a typical propeller pump when priming a siphon system and during normal operation.

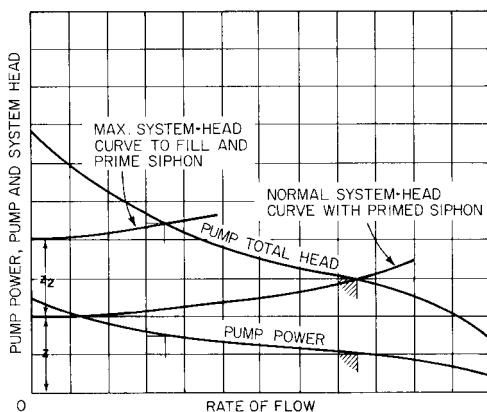


FIGURE 28 Transient system total head priming a siphon

When a pump and driver are to be selected to prime a siphon system, it is necessary to estimate the pump head and the power required to produce the minimum flow needed to start the siphon.

The minimum flow required increases with the length and the diameter and decreases with the slope of the down-leg pipe.<sup>2</sup> Prior to the removal of all the air in the system, the pump is required to provide head to raise the liquid up to and over the siphon crest. Head above the crest is required to produce a minimum flow similar to flow over a broad-crested weir. This weir head may be an appreciable part of the total pump head if the pump is low-head and large-capacity. A conservative estimate of the pump head would include a full conduit above the siphon crest. In reality, the down leg must flow partially empty before it can flow full, and it is accurate enough to estimate that the depth of liquid above the siphon crest is at critical depth for the cross section. Table 1 can be used to estimate critical depth in circular pipes, and Figure 29 can be used to calculate the cross-sectional area of the filled pipe to determine the velocity at the siphon crest.

Until all the air is removed and all the piping becomes filled, the down leg is not part of the pumping system, and its frictional and minor losses are not to be added to the maximum system-head curve to fill and prime the siphon shown in Figure 28. The total head  $TH$  in feet (meters) to be produced by the pump in Figure 27 until the siphon is primed is

$$TH = Z_3 + h_{f(1-S)} + \frac{V_c^2}{2g} \quad (13)$$

where  $Z_3$  = distance between suction level and centerline liquid at siphon crest, ft (m)

$h_{f(1-s)}$  = frictional and minor losses from 1 to  $S$ , ft (m)

$V_c^2/2g$  = velocity head at crest using actual liquid depth, approx. critical depth, ft (m)

Use of Eq. 13 permits plotting the maximum system-head curve to fill and prime the siphon for different flow rates. The pump priming flow is the intersection of the pump total head curve and this system-head curve. The pump selected must have a driver with power as shown in Figure 28 to prime the system during this transient condition.

For the pumping system shown in Figure 27, after the system is primed, the pump total head reduces to

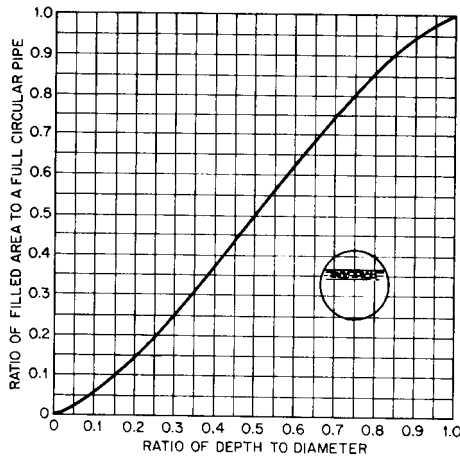
$$TH = Z + h_{f(1-2)} \quad (14)$$

**TABLE 1** Values for determining pipe-diameter ratio versus  $(\text{ft}^3/\text{s})/d^{5/2}$  in circular pipes

$\frac{D_{\text{crit}}}{d}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	...	0.0006	0.0025	0.0055	0.0098	0.0153	0.0220	0.0298	0.0389	0.0491
0.1	0.0605	0.0731	0.0868	0.1016	0.1176	0.1347	0.1530	0.1724	0.1928	0.2144
0.2	0.2371	0.2609	0.2857	0.3116	0.3386	0.3666	0.3957	0.4259	0.4571	0.4893
0.3	0.523	0.557	0.592	0.628	0.666	0.704	0.743	0.784	0.825	0.867
0.4	0.910	0.955	1.000	1.046	1.093	1.141	1.190	1.240	1.291	1.343
0.5	1.396	1.449	1.504	1.560	1.616	1.674	1.733	1.792	1.853	1.915
0.6	1.977	2.041	2.106	2.172	2.239	2.307	2.376	2.446	2.518	2.591
0.7	2.666	2.741	2.819	2.898	2.978	3.061	3.145	3.231	3.320	3.411
0.8	3.505	3.602	3.702	3.806	3.914	4.023	4.147	4.272	4.406	4.549
0.9	4.70	4.87	5.06	5.27	5.52	5.81	6.18	6.67	7.41	8.83

All tabulated values are in units of  $(\text{ft}^3/\text{s})/d^{5/2}$ ;  $d$  = diameter, ft;  $D_{\text{crit}}$  = critical depth, ft. For SI units, multiply  $(\text{m}^3/\text{h})/\text{m}^{5/2}$  by  $5.03 \times 10^{-4}$  to obtain  $(\text{ft}^3/\text{s})/\text{ft}^{5/2}$ . Examples 2, 3, and 4 illustrate the use of this table.

Source: Reference 13.



**FIGURE 29** Area versus depth for a circular pipe

In SI units:  $R = \frac{\rho V D}{\mu}$  ( $\rho$  in  $\text{kg/m}^3$ ,  $V$  in  $\text{m/s}$ ,  $D$  in  $\text{m}$ ,  $\mu$  in  $\text{N}\cdot\text{s/m}^2$ )  
 1 meter = 3.28 feet  
 1  $VD$  ( $V$  in  $\text{m/s}$ ,  $D$  in  $\text{m}$ ) = 129.2  $VD''$  ( $V$  in  $\text{ft/s}$ ,  $D''$  in inches)

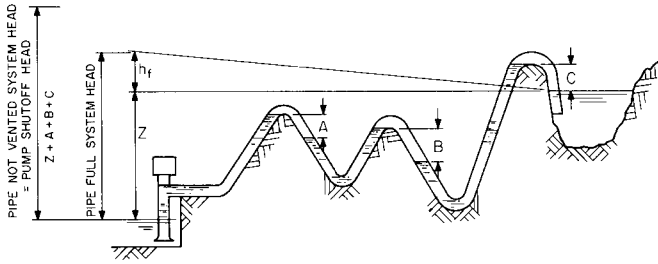
where  $Z$  = distance between suction and seal well levels (or discharge pool level if no seal well is used), ft (m)

$h_{f(1-2)}$  = frictional and minor losses from 1 to 2 (or 3 if no seal well is used), including exit velocity head loss at 2 (or 3), ft (m)

Use of Eq. 14 permits plotting of the normal system-head curve with a primed siphon, shown in Figure 28, for different flow rates. The normal pump flow is the intersection of the pump total head curve and the normal system-head curve.

If the pump cannot provide sufficient flow to prime the siphon, or if the driver does not have adequate power, the system must be primed externally by a vacuum or jet pump. An auxiliary priming pump can also be used to continuously vent the system because it is necessary that this be done to maintain full siphon recovery. In some systems, the water pumped is saturated with air, and as the liquid flows through the system, the pressure is reduced (and in cooling systems the temperature is increased). Both these conditions cause the release of some of the entrained air. Air will accumulate at the top of the siphon and in the upper parts of the down leg. The siphon works on the principle that an increase in elevation in the up leg produces a decrease in pressure and an equal decrease in elevation in the down leg resulting in recovery of this pressure. This cannot occur if the density of the liquid in the down leg is decreased as a result of the formation of air pockets. These air pockets also restrict the flow area. A release of entrained air and air leakage into the system through pipe joints and fittings will result in a centrifugal pump's delivering less than design flow, as the head will be higher than estimated. Also, high-specific-speed pumps with rising power curves toward shutoff can become overloaded. In order to maintain full head recovery, it is necessary to continuously vent the siphon at the top and at several points along the down leg, especially at the beginning of a change in slope.<sup>3</sup> These venting points can be manifolded together and connected to a single downward venting system.

Some piping systems may contain several up and down legs; that is, several siphons in series. Each down leg, as in a single siphon, is vulnerable to air or vapor binding. The likelihood of flow reduction, and conceivably in some cases complete flow shutoff, is increased in a multiple siphon system.<sup>3</sup> As shown in Figure 30, system static head is increased if



**FIGURE 30** Multiple siphon system ( $Z$  = normal system static head when pipe is flowing full). Pump flow is stopped when normal static head plus sum of air pocket heights equals pump shutoff head.

proper venting is not provided at the top of each siphon. Normally the static head is the difference between outlet and inlet elevations. If air pockets exist, head cannot be recovered and the normal static head is increased by the sum of the heights of all the intermediate liquidless pockets. Flow will stop when the total static head equals the pump shutoff head.

The following examples illustrate the use of Eqs. 3, 9, 12, 13, and 14, Table 1, and Figure 29.

**EXAMPLE 2** A pump is required to produce a flow of 70,000 gpm (15,900 m<sup>3</sup>/h) through the system shown in Figure 27. Calculate the system total head from point 1 to point 3 (no seal well) under the following conditions:

Specific gravity = 0.998 for 80°F (26.7°C) water

Barometric pressure = 29 in (73.7 cm) mercury abs (sp. gr. 13.6)

Suction and discharge water levels are equal,  $Z = 0$

$Z_1 = 40$  ft (12.2 m)

$H_{f(1-s)} = 3$  ft (0.91 m) up-leg frictional head

$h_{f(s-3)} = 3.3$  ft (1 m) down-leg frictional head, including exit loss

Pipe diameter = 48 in (121.9 cm) ID

Water vapor pressure = 0.507 lb/in<sup>2</sup> (3.5 kPa) abs at 80°F (26.7°C)

The maximum siphon height may be found from Eq. 12:

$$Z_s = H_B + h_{f(s-3)} - H_S + \frac{V_S^2}{2g}$$

From Eq. 3

$H_B$  = barometric pressure head in feet (meters) of liquid pumped

In USCS units  $\frac{\text{sp. gr.}_1}{\text{sp. gr.}_2} h_1 = \frac{13.6}{0.998} \times \frac{29}{12} = 32.9$  ft abs

$$H_S = \frac{p}{\gamma} = \frac{144 \times 0.507}{62.19} = 1.17$$
 ft abs

In SI units  $\frac{\text{sp. gr.}_1}{\text{sp. gr.}_2} h_1 = \frac{13.6}{0.998} \times \frac{73.7}{100} = 10$  m abs

$$H_S = \frac{p}{\gamma} = \frac{3.5 \times 1000}{9769} = 0.358$$
 m abs

From Eqs. 9 and 12,

$$\text{in USCS units } V_s = \frac{\text{gpm}}{(\text{pipe ID in inches})^2} \times 0.408 = \frac{70,000}{48^2} \times 0.408 = 12.4 \text{ ft/s}$$

$$Z_s = 32.9 + 3.3 - 1.17 - \frac{12.4^2}{2 \times 32.17} = 32.63 \text{ ft}$$

$$\text{in SI units } V_s = \frac{\text{m}^3/\text{h}}{(\text{pipe ID in cm})^2} \times 3.54 = \frac{15,900}{121.9^2} \times 3.54 = 3.79 \text{ m/s}$$

$$Z_s = 10 + 1 - 0.358 - \frac{3.79^2}{2 \times 9.807} = 9.1 \text{ m}$$

Because the maximum height is exceeded ( $Z_1 > Z_s$ ), siphon recovery is not possible. The system total head is therefore found from Eq. 13:

$$TH = Z_3 + h_{f(1-s)} + \frac{V_c^2}{2g}$$

The critical depth  $D_{\text{crit}}$  is found using Table 1:

$$\text{In USCS units } \frac{\text{ft}^3/\text{s}}{7.481 \times 60} = \frac{70,000}{60} = 156$$

$$\frac{\text{ft}^3/\text{s}}{d^{5/2}} = \frac{156}{4^{5/2}} = 4.88$$

$$\frac{D_{\text{crit}}}{d} = 0.901$$

$$D_{\text{crit}} = 0.901 \times 4 = 3.6 \text{ ft}$$

$$\text{In SI units } \frac{\text{m}^3/\text{h}}{d^{5/2}} = \frac{15,900}{1.219^{5/2}} = 9695$$

Convert to USCS units (see footnote to Table 1):

$$\frac{\text{ft}^3/\text{s}}{\text{ft}^{5/2}} = 9695 \times 5.03 \times 10^{-4} = 4.88$$

$$\frac{D_{\text{crit}}}{d} = 0.901$$

$$D_{\text{crit}} = 0.901 \times 1.219 = 1.1 \text{ m}$$

To calculate the water velocity at the siphon crest, determine the area of the filled pipe. From Figure 29, ratio of filled area to area of a full pipe is 0.95 for a depth-to-diameter ratio of 0.901:

$$\text{In USCS units } V_c = \frac{\text{gpm}}{0.95(\text{ID in inches})^2} \times 0.408 = \frac{70,000}{0.95 \times 48^2} \times 0.408 = 13.0 \text{ ft/s}$$

$$Z_3 = 40 - 4 + \frac{3.6}{2} = 37.8 \text{ ft}$$

$$\text{From Eq. 13 } TH = 37.8 + 3 + \frac{13.0^2}{2 \times 32.17} = 43.43 \text{ ft}$$

$$\text{In SI units } V_c = \frac{\text{m}^3/\text{h}}{0.95(\text{ID in cm})^2} \times 3.54 = \frac{15,900}{0.95 \times 121.9^2} \times 3.54 = 3.99 \text{ m/s}$$

$$Z_3 = 12.2 - 1.219 + \frac{1.1}{2} = 11.53 \text{ m}$$

$$\text{From Eq. 13} \quad TH = 11.53 + 0.91 + \frac{3.99^2}{2 \times 9.807} = 13.25 \text{ m}$$

EXAMPLE 3 Calculate the minimum total system head using conditions in Example 2 and a seal well, as shown in Figure 27. Use 2.8 ft (0.853 m) for the frictional head loss  $h_{f(s-2)}$ .

The maximum siphon height  $Z_s$  in Example 2 was found to be 32.63 ft (9.95 m). Therefore from Eq. 14 the total system head after priming is

$$TH = Z + h_{f(1-2)}$$

$$\text{In USCS units} \quad h_{f(1-2)} = h_{f(1-S)} + h_{f(S-2)} = 3 + 2.8 = 5.8 \text{ ft}$$

$$Z = Z_1 - Z_s = 40 - 32.63 = 7.37 \text{ ft}$$

$$\text{In SI units} \quad h_{f(1-2)} = h_{f(1-S)} + h_{f(S-2)} = 0.91 + 0.853 = 1.763 \text{ ft}$$

$$Z = Z_1 - Z_s = 12.2 - 9.95 = 2.25 \text{ m}$$

Note that the seal well elevation is above discharge level. Therefore

$$\text{in USCS units} \quad TH = 7.37 + 5.8 = 13.17 \text{ ft}$$

$$\text{in SI units} \quad TH = 2.25 + 1.763 = 4.01 \text{ m}$$

EXAMPLE 4 The dimensions of the down leg in Example 3 require a minimum velocity of 5 ft/s (1.52 m/s) flowing full to purge air from the system and start the siphon. Calculate the system head the pump must overcome to prime the siphon.

$$\text{In USCS units} \quad \text{gpm} = \frac{V(\text{pipe ID in inches})^2}{0.408} = \frac{5 \times 48^2}{0.408} = 28,200$$

$$\text{ft}^3/\text{s} = 28,200 \div 449 = 62.8$$

$$\text{In SI units} \quad \text{m}^3/\text{h} = \frac{V(\text{pipe ID in cm})^2}{3.54} = \frac{1.52 \times 121.9^2}{3.54} = 6400$$

The critical depth is found from Table 1:

$$\text{In USCS units} \quad \frac{\text{ft}^3/\text{s}}{d^{5/2}} = \frac{62.8}{4^{5/2}} = 1.97$$

$$\frac{D_{\text{crit}}}{d} = 0.6$$

$$D_{\text{crit}} = 0.6 \times 4 = 2.4 \text{ ft}$$

$$\text{In SI units} \quad \frac{\text{m}^3/\text{h}}{d^{5/2}} = \frac{6400}{1.219^{5/2}} = 3902$$

Convert to USCS units (see footnote to Table 1):

$$\frac{\text{ft}^3/\text{s}}{d^{5/2}} = 3902 \times 5.03 \times 10^{-4} = 1.97$$

$$\frac{D_{\text{crit}}}{d} = 0.6$$

$$D_{\text{crit}} = 0.6 \times 1.219 = 0.73 \text{ m}$$

From Figure 29, the ratio of the filled area to the area of a full pipe is 0.625 for a depth-to-diameter ratio of 0.60:

$$\text{In USCS units} \quad V_c = \frac{5}{0.625} = 8.0 \text{ ft/s}$$

$$\text{In SI units} \quad V_c = \frac{1.52}{0.625} = 2.4 \text{ m/s}$$

$$\text{From Eq. 13} \quad TH = Z_3 + h_{f(1-S)} + \frac{V_c^2}{2g}$$

$$\text{In USCS units} \quad Z_2 = Z_S - 4 + \frac{D_{\text{crit}}}{2} = 32.63 - 4 + \frac{2.4}{2} = 29.83 \text{ ft}$$

$$Z = 7.37 \text{ ft (from Example 3)}$$

$$h_{f(1-S)} \text{ at } 28,200 \text{ gpm} = \left( \frac{28,200}{70,000} \right)^2 \times 3 = 0.49 \text{ ft}$$

$$TH = (7.37 + 29.83) + 0.49 + \frac{8^2}{2 \times 32.17} = 38.68 \text{ ft}$$

$$\text{In SI units} \quad Z_2 = Z_S - 1.219 + \frac{D_{\text{crit}}}{2} = 9.95 - 1.219 + \frac{0.73}{2} = 9.1 \text{ m}$$

$$Z = 2.25 \text{ m (from Example 3)}$$

$$h_{f(1-S)} \text{ at } 6400 \text{ m}^3/\text{h} = \left( \frac{6400}{15,900} \right)^2 \times 0.91 = 0.15 \text{ m}$$

$$TH = (2.25 + 9.1) + 0.15 + \frac{2.4^2}{2 \times 9.807} = 11.79 \text{ m}$$

If the system is not externally primed, the centrifugal pump selected must be able to deliver at least 28,000 gpm (6400 m<sup>3</sup>/h) at 38.68 ft (11.79 m) total head and must be provided with a driver having adequate power for this condition. After the system is primed, the pump must be capable of delivering at least 70,000 gpm (15,900 m<sup>3</sup>/h) at 13.17 ft (4.01 m) (see Figure 28).

## HEAD LOSSES IN SYSTEM COMPONENTS

**Pressure Pipes** Resistance to flow through a pipe is caused by viscous shear stresses in the liquid and by turbulence at the pipe walls. *Laminar* flow occurs in a pipe when the average velocity is relatively low and the energy head is lost mainly as a result of viscosity. In laminar flow, liquid particles have no motion next to the pipe walls and flow occurs as a result of the movement of particles in parallel lines with velocity increasing toward the center. The movement of concentric cylinders past each other causes viscous shear stresses, more commonly called *friction*. As flow increases, the flow pattern changes, the average velocity becomes more uniform, and there is less viscous shear. As the laminar film decreases in thickness at the pipe walls and as the flow increases, the pipe roughness becomes important because it causes turbulence. *Turbulent* flow occurs when average pipe velocity is relatively high and energy head is lost predominantly because of turbulence caused by the wall roughness. The average velocity at which the flow changes from laminar to turbulent is not definite, and there is a critical zone in which either laminar or turbulent flow can occur.

Viscosity can be visualized as follows. If the space between two planar surfaces is filled with a liquid, a force will be required to move one surface at a constant velocity relative to the other. The velocity of the liquid will vary linearly between the surfaces. The ratio of the force per unit area, called *shear stress*, to the velocity per unit distance between surfaces, called *shear* or *deformation rate*, is a measure of a liquid's *dynamic* or *absolute viscosity*.

Liquids such as water and mineral oil, which exhibit shear stresses proportional to shear rates, have a constant viscosity for a particular temperature and pressure and are called *Newtonian* or *true liquids*. In the normal pumping range, however, the viscosity of true liquids may be considered independent of pressure. For these liquids, the viscosity remains constant because the rate of deformation is directly proportional to the shearing stress. The viscosity and resistance to flow, however, increase with decreasing temperature.

Liquids such as molasses, grease, starch, paint, asphalt, and tar behave differently from Newtonian liquids. The viscosity of the former does not remain constant and their shear, or deformation, rate increases more than the stress increases. These liquids, called *thixotropic*, exhibit lower viscosity as they are agitated at a constant temperature.

Still other liquids, such as mineral slurries, show an increase in viscosity as the shear rate is increased and are called *dilatant*.

In USCS units, dynamic (absolute) viscosity is measured in pound-seconds per square foot or slugs per foot-second. In SI measure, the units are newton-seconds per square meter or pascal-seconds. Usually dynamic viscosity is measured in *poises* ( $1 \text{ P} = 0.1 \text{ Pa} \cdot \text{s}$ ) or in *centipoises* ( $1 \text{ cP} = \frac{1}{100} \text{ P}$ ):

$$1 \text{ lb} \cdot \text{s}/\text{ft}^2 = 47.8801 \text{ Pa} \cdot \text{s} = 47,880.1 \text{ cP}$$

The viscous property of a liquid is also sometimes expressed as *kinematic viscosity*. This is the dynamic viscosity divided by the mass density (specific weight/ $g$ ). In USCS units, kinematic viscosity is measured in square feet per second. In SI measure, the units are square meters per second. Usually kinematic viscosity is measured in *stokes* ( $1 \text{ St} = 0.0001 \text{ m}^2/\text{s}$ ) or in *centistokes* ( $1 \text{ cSt} = \frac{1}{100} \text{ St}$ ):

$$1 \text{ ft}^2/\text{s} = 0.0929034 \text{ m}^2/\text{s} = 92,903.4 \text{ cSt}$$

A common unit of kinematic viscosity in the United States is Saybolt seconds universal (SSU) for liquids of medium viscosity and Saybolt seconds Furol (SSF) for liquids of high viscosity. Viscosities measured in these units are determined by using an instrument that measures the length of time needed to discharge a standard volume of the sample. Water at 60°F (15.6°C) has a kinematic viscosity of approximately 31 SSU (1.0 cSt). For values of 70 cSt and above,

$$\text{cSt} = 0.216 \text{ SSU}$$

$$\text{SSU} = 10 \text{ SSF}$$

The dimensionless Reynolds number  $Re$  is used to describe the type of flow in a pipe flowing full and can be expressed as follows:

$$Re = \frac{VD}{v} = \frac{\rho VD}{\mu} \quad (15)$$

where  $V$  = average pipe velocity, ft/s (m/s)

$D$  = inside pipe diameter, ft (m)

$v$  = liquid kinematic viscosity, ft<sup>2</sup>/s (m<sup>2</sup>/s)

$\rho$  = liquid density, slugs/ft<sup>3</sup> (kg/m<sup>3</sup>)

$\mu$  = liquid dynamic (or absolute) viscosity slug/ft · s (N · s/m<sup>2</sup>)

*Note:* The dimensionless Reynolds number is the same in both USCS and SI units.

When the Reynolds number is 2000 or less, the flow is generally laminar, and when it is greater than 4000, the flow is generally turbulent. The Reynolds number for the flow of water in pipes is usually well above 4000, and therefore the flow is almost always turbulent.

The Darcy-Weisbach formula is the one most often used to calculate pipe friction. This formula recognizes that friction increases with pipe wall roughness, with wetted surface area, with velocity to a power, and with viscosity and decreases with pipe diameter to a power and with density. Specifically, the frictional head loss  $h_f$  in feet (meters) is



$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (16)$$

where  $f$  = friction factor

$L$  = pipe length, ft (m)

$D$  = inside pipe diameter, ft (m)

$V$  = average pipe velocity, ft/s (m/s)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

For laminar flow, the friction factor  $f$  is equal to  $64/Re$  and is independent of pipe wall roughness. For turbulent flow,  $f$  for all incompressible fluids can be determined from the well-known Moody diagram, shown in Figure 31. To determine  $f$ , it is required that the Reynolds number and the relative pipe roughness be known. Values of relative roughness  $(\epsilon/D)$ , where  $\epsilon$  is a measure of pipe wall roughness height in feet (meters), can be obtained from Figure 32 for different pipe diameters and materials. Figure 32 also gives values for  $f$  for the flow of 60°F (15.6°C) water in rough pipes with complete turbulence. Values of kinematic viscosity and Reynolds numbers for a number of different liquids at various temperatures are given in Figure 33. The Reynolds numbers of 60°F (15.6°C) water for various velocities and pipe diameters may be found by using the  $VD''$  scale in Figure 31.

There are many empirical formulas for calculating pipe friction for water flowing under turbulent conditions. The most widely used is the Hazen-Williams formula:

$$\text{In USCS units} \quad V = 1.318 C r^{0.63} S^{0.54} \quad (17a)$$

$$\text{In SI units} \quad V = 0.8492 C r^{0.63} S^{0.54} \quad (17b)$$

where  $V$  = average pipe velocity, ft/s (m/s)

$C$  = friction factor for this formula, which depends on roughness only

$r$  = hydraulic radius (liquid area divided by wetted perimeter) or  $D/4$  for a full pipe, ft (m)

$S$  = hydraulic gradient or frictional head loss per unit length of pipe, ft/ft (m/m)

The effect of age on a pipe should be taken into consideration when estimating the frictional loss. A lower  $C$  value should be used, depending on the expected life of the system. Table 2 gives recommended friction factors for new and old pipes. A value of  $C$  of 150 may be used for plastic pipe. Figure 34 is a nomogram that can be used in conjunction with Table 2 for a solution to the Hazen-Williams formula.

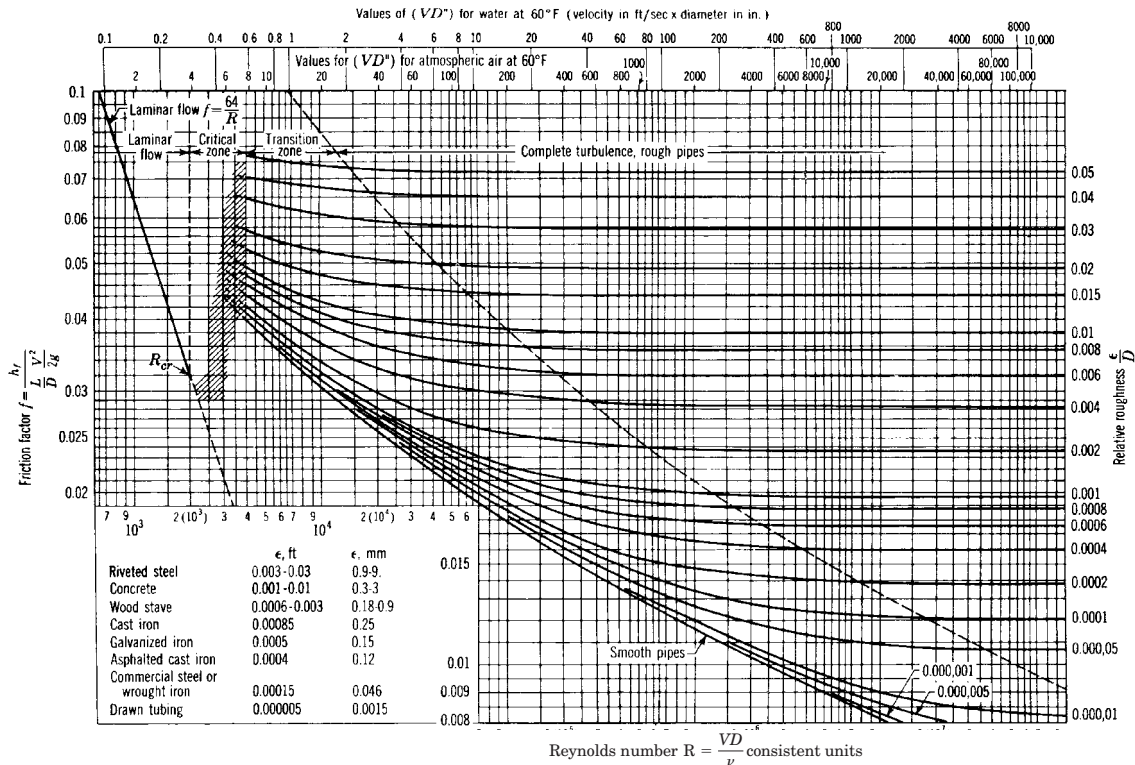
The frictional head loss in pressure pipes can be found by using either the Darcy-Weisbach formula (Eq. 16) or the Hazen-Williams formula (Eq. 17). Tables in the appendix give Darcy-Weisbach friction values for Schedule 40 new steel pipe carrying water. Tables are also provided for losses in old cast iron piping based on the Hazen-Williams formula with  $C = 100$ . In addition, values of  $C$  for various pipe materials, conditions, and years of service can also be found in the appendix.

The following examples illustrate how Figures 31, 32, and 33 and Table 2 may be used.

**EXAMPLE 5** Calculate the Reynolds number for 175°F (79.4°C) kerosene flowing through 4-in (10.16-cm), Schedule 40, 3.426-in (8.70-cm) ID, seamless steel pipe at a velocity of 14.6 ft/s (4.45 m/s).

$$\text{In USCS units} \quad VD'' = 14.6 \times 3.426 = 50 \text{ ft/s} \times \text{in}$$

$$\text{In SI units} \quad VD = 4.45 \times 0.087 = 0.387 \text{ m/s} \times \text{m} = 0.387 \times 129.2 = 50 \text{ ft/s} \times \text{in}$$



**FIGURE 31** Moody diagram. (Reference 14) In SI units:  $R = \frac{\rho VD}{\mu}$  ( $\rho$  in  $\text{kg/m}^3$ ,  $V$  in  $\text{m/s}$ ,  $D$  in  $\text{m}$ ,  $\mu$  in  $\text{N} \cdot \text{s/m}^2$ .) 1 meter = 3.28 ft;  $1 \text{VD}$  ( $V$  in  $\text{m/s}$ ,  $D$  in  $\text{m}$ ) =  $129.2 \text{VD}''$  ( $V$  in  $\text{ft/s}$ ,  $D''$  in inches).

**TABLE 2** Values of friction factor *C* to be used with the Hazen-Williams formula in Figure 34

Type of pipe	Age	Size, in <sup>a</sup>	<i>C</i>
Cast iron	New	All sizes	130
		12 and over	120
		8	119
		4	118
	10 years old	24 and over	113
		12	111
		4	107
	20 years old	24 and over	100
		12	96
		4	89
	30 years old	30 and over	
		16	87
		4	75
	40 years old	30 and over	83
		16	80
		4	64
Welded steel	Any age, any size	40 and over	77
		24	74
Riveted steel	Any age, any size	4	55
			Same as for cast iron pipe
Wood-stave	Average value, regardless of age and size		5 years old
			10 years older
			120
			140
Concrete or concrete-lined	Large sizes, good workmanship, steel forms		120
			135
Vitrified	Centrifugally spun		110
			110

<sup>a</sup>In × 25.4 = mm  
Source: Adapted From Reference 15.

Follow the tracer lines in Figure 33 and read directly:

$$Re = 3.5 \times 10^5$$

**EXAMPLE 6** Calculate the frictional head loss for 100 ft (30.48 m) of 20-in (50.8-cm), Schedule 20, 19.350-in (49.15-cm) ID, seamless steel pipe for 109°F (42.8°C) water flowing at a rate of 11,500 gpm (2612 m³/h). Use the Darcy-Weisbach formula.

In USCS Units

$$V = \frac{\text{gpm}}{(\text{pipe ID in inches})^2} \times 0.408 = \frac{11,500}{19.35^2} \times 0.408 = 12.53 \text{ ft/s}$$
$$VD'' = 12.53 \times 19.35 = 242 \text{ ft/s} \times \text{in}$$

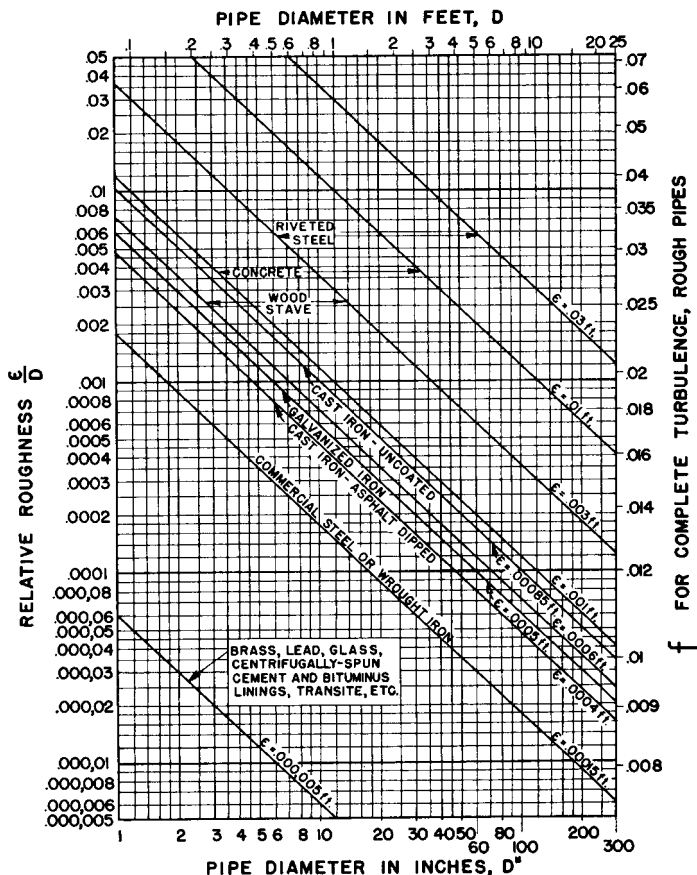


FIGURE 32 Relative roughness and friction factors for new, clean pipes for flow of 60°F (15.6°C) water (Hydraulic Institute Engineering Data Book, Reference 5) (1 meter 39.37 in = 3.28 ft).

In SI units

$$V = \frac{\text{m}^3/\text{h}}{(\text{pipe ID in cm})} \times 3.54 = \frac{2612}{49.15^2} \times 3.54 = 3.83 \text{ m/s}$$

$$VD = 3.83 \times 0.4915 = 1.88 \text{ m/s} \times \text{m} = 1.88 \times 129.2 = 242 \text{ ft/s} \times \text{in}$$

From Figure 33

$$Re = 3 \times 10^6$$

From Figure 32

$$\frac{\epsilon}{D} = 0.00009$$

From Figure 31

$$f = 0.012$$

Using Eq. 16,

In USCS units

$$D = \frac{19.35}{12} = 1.61 \text{ ft}$$



$$h_f = f \frac{L}{d} \frac{V^2}{2g} = 0.012 \frac{100}{1.61} \times \frac{12.53^2}{2 \times 32.17} = 1.82 \text{ ft}$$

$$\text{In SI units} \quad h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.012 \frac{30.48}{0.4915} \times \frac{3.83^2}{2 \times 9.807} = 0.556 \text{ m}$$

**EXAMPLE 7** The flow in Example 6 is increased until complete turbulence results. Determine the friction factor  $f$  and flow.

From Figure 31, follow the relative roughness curve  $\varepsilon/D = 0.00009$  to the beginning of the zone marked “complete turbulence, rough pipes” and read

$$f = 0.0119 \quad \text{at } Re = 2 \times 10^7$$

The problem may also be solved using Figure 32. Enter relative roughness  $\varepsilon/D = 0.00009$  and read directly across to

$$f = 0.0119$$

An increase in  $Re$  from  $3 \times 10^6$  to  $2 \times 10^7$  would require an increase in flow to

$$\text{in USCS units} \quad \frac{2 \times 10^7}{3 \times 10^6} \times 11,500 = 76,700 \text{ gpm}$$

$$\text{in SI units} \quad \frac{2 \times 10^7}{3 \times 10^6} \times 2612 = 17,413 \text{ m}^3/\text{h}$$

**EXAMPLE 8** The liquid in Example 6 is changed to water at 60°F (15.6°C). Determine  $Re$ ,  $f$ , and the frictional head loss per 100 ft (100 m) of pipe.

$$VD'' = 242 \text{ ft/s} \times \text{in} \quad (\text{as in Example 6})$$

Because the liquid is 60°F (15.6°C) water, enter Figure 31 and read directly downward from  $VD''$  to

$$Re = 1.8 \times 10^6$$

Where the line  $VD''$  to  $Re$  crosses  $\varepsilon/D = 0.00009$  in Figure 31, read

$$f = 0.013$$

Water at 60°F (15.6°C) is more viscous than 109°F (42.8°C) water, and this accounts for the fact that  $Re$  decreases and  $f$  increases. Using Eq. 16, it can be calculated that the frictional head loss increases to

$$\text{in USCS units} \quad h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.013 \frac{100}{1.61} \times \frac{12.53^2}{2 \times 32.17} = 1.97 \text{ ft}$$

$$\text{in SI units} \quad h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.013 \frac{100}{0.4915} \times \frac{3.83^2}{2 \times 9.807} = 1.97 \text{ m}$$

**EXAMPLE 9** A 102-in (259-cm) ID welded steel pipe is to be used to convey water at a velocity of 11.9 ft/s (3.63 m/s). Calculate the expected loss of head due to friction per 1000 ft and per 1000 m of pipe after 20 years. Use the empirical Hazen-Williams formula.

From Table 2,  $C = 100$ .

$$\text{In USCS units} \quad r = \frac{D}{4} = \frac{102}{(4 \times 12)} = 2.13 \text{ ft}$$

$$\text{In SI units} \quad r = \frac{D}{4} = \frac{2.59}{4} = 0.648 \text{ m}$$

Substituting in Eq. 17,

$$\begin{aligned} \text{in USCS units} \quad S^{0.54} &= \frac{V}{1.318Cr^{0.63}} = \frac{11.9}{1.318 \times 100 \times 2.13^{0.63}} = 0.0557 \\ S &= (0.0557)^{1/0.54} = 0.0048 \text{ ft/ft} \\ h_f &= 1000 \times 0.0048 = 4.8 \text{ ft} \end{aligned}$$

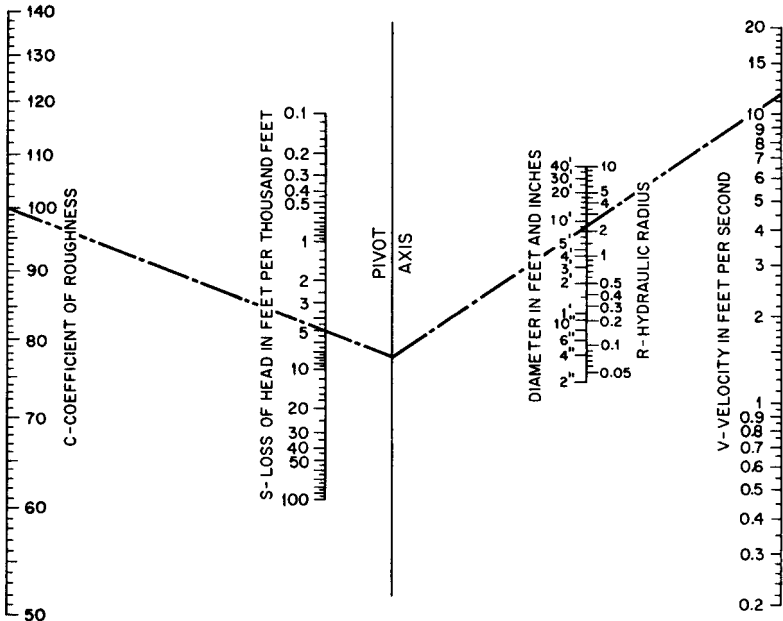
$$\begin{aligned} \text{in SI units} \quad S^{0.54} &= \frac{V}{0.8492Cr^{0.63}} = \frac{3.63}{0.8492 \times 100 \times 0.648^{0.63}} = 0.0562 \\ S &= (0.0562)^{1/0.54} = 0.0048 \text{ m/m} \\ h_f &= 1000 \times 0.0048 = 4.8 \text{ m} \end{aligned}$$

The problem may also be solved by using Figure 34, following the trace lines:

$$h_f \approx 5 \text{ ft (m)}$$

**Frictional Loss for Viscous Liquids** Table 3 gives the frictional loss for viscous liquids flowing in new Schedule 40 steel pipe. Values of pressure loss are given for both laminar and turbulent flows.

For laminar flow, the pressure loss is directly proportional to the viscosity and the velocity of flow and inversely proportional to the pipe diameter to the fourth power. Therefore, for intermediate values of viscosity and flow, obtain the pressure loss by direct inter-



**FIGURE 34** Nomogram for the solution of the the Hazen-Williams formula. Obtain values for  $C$  from Table 2. (Reference 15) (1 m/s = 3.28 ft/s; 1 m = 39.37 in)

polation. For pipe sizes not shown, multiply the fourth power of the ratio of any tabulated diameter to the pipe diameter wanted by the tabulated loss shown. The flow rate and viscosity must be the same for both diameters.

For turbulent flow and for rates of flow and pipe sizes not tabulated, the following procedures may be followed. For the viscosity and pipe size required, an intermediate flow loss is found by selecting the pressure loss for the next lower flow and multiplying by the square of the ratio of actual to tabulated flow rates. For the viscosity and flow required, an intermediate pipe diameter flow loss is found by selecting the pressure loss for the next smaller diameter and multiplying by the fifth power of the ratio of tabulated to actual inside diameters.

The viscosity of various common liquids can be found in tables in the appendix.

**Partially Full Pipes and Open Channels** Another popular empirical equation applicable to the flow of water in pipes flowing full or partially full or in open channels is the Manning formula:

$$\text{In USCS units} \quad V = \frac{1.486}{n} r^{2/3} S^{1/2} \quad (18a)$$

$$\text{In SI units} \quad V = \frac{r^{2/3} S^{1/2}}{n} \quad (18b)$$

where  $V$  = average velocity, ft/s (m/s)

$n$  = friction factor for this formula, which depends on roughness only

$r$  = hydraulic radius (liquid area divided by wetted perimeter), ft (m)

$S$  = hydraulic gradient or frictional head loss per unit length of conduit, ft/ft (m/m)

The Manning formula nomogram shown in Figure 35 can be used to determine the flow or frictional head loss in open or closed conduits. Note that the hydraulic gradient  $S$  in Figure 35 is plotted in feet per 100 ft of conduit length. Values of friction factor  $n$  are given in Table 4.

If the conduit is flowing partially full, computing the hydraulic radius is sometimes difficult. When the problem to be solved deals with a pipe that is not flowing full, Figure 36 may be used to obtain multipliers for correcting the flow and velocity of a full pipe to the values needed for the actual fill condition. If the flow in a partially full pipe is known and the frictional head loss is to be determined, Figure 36 is first used to correct the flow to what it would be if the pipe were full. Then Eq. 18 or Figure 35 is used to determine the frictional head loss (which is also the hydraulic gradient and the slope of the pipe). The problem is solved in reverse if the hydraulic gradient is known and the flow is to be determined.

For full or partially full flow in conduits that are not circular in cross section, an alternate solution to using Eq. 18 is to calculate an equivalent diameter equal to four times the hydraulic radius. If the conduit is extremely narrow and width is small relative to length (annular or elongated sections), the hydraulic radius is one-half the width of the section.<sup>4</sup> After the equivalent diameter has been determined, the problem may be solved by using the Darcy-Weisbach formula (Eq. 16).

The hydraulic gradient in a uniform open channel is synonymous with frictional head loss in a pressure pipe. The hydraulic gradient of an open channel or of a pipe flowing partially full is the slope of the free liquid surface. In the reach of the channel where the flow is uniform, the hydraulic gradient is parallel to the slope of the channel bottom. Figure 37 shows that, in a pressure pipe of uniform cross section, the slope of both the energy and hydraulic gradients is a measure of the frictional head loss per foot (meter) of pipe between points 1 and 2. Figure 38 illustrates the flow in an open channel of varying slope. Between points 1 and 2, the flow is uniform and the liquid surface (hydraulic gradient) and channel bottom are both parallel and their slope is the frictional head loss per foot (meter) of channel length.



**TABLE 3** Frictional loss for viscous liquids (Hydraulic Institute Engineering Data Book, Reference 5)

gpm	Pipe size	Viscosity, SSU							
		100	200	300	400	500	1000	1500	2000
3	$\frac{1}{2}$	11.2	23.6	35.3	47.1	59	118	177	236
	$\frac{3}{4}$	3.7	7.6	11.5	15.3	19.1	38.2	57	76
	1	1.4	2.9	4.4	5.8	7.3	14.5	21.8	29.1
5	$\frac{3}{4}$	6.1	12.7	19.1	25.5	31.9	61	96	127
	1	2.3	4.9	7.3	9.7	12.1	21.2	36.3	48.5
	$1\frac{1}{4}$	0.77	1.6	2.4	3.3	4.1	8.1	12.2	16.2
7	$\frac{3}{4}$	8.5	17.9	26.8	35.7	44.6	89	134	178
	1	3.2	6.8	10.2	13.6	17	33.9	51	68
	$1\frac{1}{4}$	1.1	2.3	3.4	4.5	5.7	11.4	17	22.7
10	1	4.9	9.7	14.5	19.4	24.2	48.5	73	97
	$1\frac{1}{4}$	1.6	3.3	4.9	6.5	8.1	16.2	24.3	32.5
	$1\frac{1}{2}$	0.84	1.8	2.6	3.5	4.4	8.8	13.1	17.5
15	1	11	14.5	21.8	29.1	36.3	73	109	145
	$1\frac{1}{4}$	2.8	4.9	7.3	9.7	12.2	24.3	36.5	48.7
	$1\frac{1}{2}$	1.3	2.6	3.9	5.3	6.6	13.1	19.7	26.3
20	1	18	18	29.1	38.8	48.5	97	145	194
	$1\frac{1}{4}$	4.9	6.4	9.7	13	16.2	32.5	48.7	65
	$1\frac{1}{2}$	2.3	3.5	5.3	7	8.8	17.5	26.3	35
	2	0.64	1.3	1.9	2.6	3.2	6.4	9.6	12.9
25	$1\frac{1}{2}$	3.5	4.4	6.6	8.8	11	21.9	32.8	43.8
	2	1	1.6	2.4	3.2	4	8	12.1	16.1
	$2\frac{1}{2}$	0.4	0.79	1.2	1.6	2	4	5.9	7.9
30	$1\frac{1}{2}$	5	5.3	7.9	10.5	13.1	26.3	39.4	53
	2	1.4	1.9	2.9	3.9	4.8	9.6	14.5	19.3
	$2\frac{1}{2}$	0.6	0.95	1.4	1.9	2.4	4.7	7.1	9.5
40	$1\frac{1}{2}$	8.5	9	10.5	14	17.5	35	53	70
	2	2.5	2.5	3.9	5.1	6.4	12.9	19.3	25.7
	$2\frac{1}{2}$	1.1	1.3	1.9	2.5	3.2	6.3	9.5	12.6
50	$1\frac{1}{2}$	12.5	14	14	17.5	21.9	43.8	66	88
	2	3.7	4	4.8	6.4	8	16.1	24.1	32.1
	$2\frac{1}{2}$	1.6	1.7	2.4	3.2	4	7.9	11.8	15.8
60	2	5	5.8	5.8	7.7	9.6	19.3	28.9	38.5
	$2\frac{1}{2}$	2.2	2.4	2.8	3.8	4.7	9.5	14.2	19
	3	0.8	0.8	1.2	1.6	2	4	6	8
70	$2\frac{1}{2}$	2.8	3.2	3.4	4.4	5.5	11.1	16.6	22.1
	3	1	1.1	1.4	1.9	2.3	4.6	7	9.3
	4	0.27	0.31	0.47	0.63	0.78	1.6	2.4	3.1
80	$2\frac{1}{2}$	3.6	4.2	4.2	5.1	6.3	12.6	19	25.3
	3	1.3	1.4	1.6	2.1	2.7	5.3	8	10.6
	4	0.36	0.36	0.54	0.72	0.89	1.8	2.7	3.6
100	$2\frac{1}{2}$	5.3	6.1	6.4	6.4	8	15.8	23.7	31.6
	3	1.9	2.2	2.2	2.7	3.3	6.6	9.9	13.3
	4	0.52	0.57	0.67	0.89	1.1	2.2	3.4	4.5

←←← TURBULENT FLOW →→→      ←←← LAMINAR FLOW →→→

Loss in pounds per square inch per 100 ft of new Schedule 40 steel pipe based on specific gravity of 1.00 of that liquid. For commercial installations, it is recommended that 15% be added to the values in this table.

TABLE 3 Continued.

Viscosity, SSU									
2500	3000	4000	5000	6000	7000	8000	9000	10,000	15,000
294	353	471	589	706	824	942	...	...	...
96	115	153	191	229	268	306	344	382	573
36.3	43.6	58	73	87	101	116	131	145	218
159	191	255	319	382	446	510	573	637	956
61	73	97	121	145	170	194	218	242	363
20.3	24.3	32.5	40.6	48.7	57	65	73	81	122
223	268	357	416	535	624	713	803	892	...
85	102	136	170	203	237	271	305	339	509
28.4	34.1	45.4	57	68	80	91	102	114	170
121	145	194	242	291	339	388	436	485	727
40.6	48.7	65	81	97	114	130	146	162	243
21.9	26.3	35	43.8	53	61	70	79	88	131
182	218	291	363	436	509	581	654	727	...
61	73	97	122	146	170	195	219	243	365
32.8	39.4	53	66	79	92	105	118	131	197
242	291	388	485	581	678	775	872	...	...
81	97	130	162	195	227	260	292	325	487
43.8	53	70	88	105	123	140	158	175	263
16.1	19.3	25.7	32.1	38.5	45	51	58	64	96
55	66	88	110	131	153	176	197	219	328
20.1	24.1	32.1	40.2	48.2	56	61	72	80	121
9.9	11.8	15.8	19.7	23.7	27.6	31.6	35.5	39.5	59
66	79	105	131	158	184	210	237	263	394
24.1	28.9	38.5	48.2	58	67	77	87	96	145
11.8	14.2	19	23.7	28.4	33.2	37.9	42.6	47.4	71
88	105	140	175	210	245	280	315	350	526
32.1	38.5	51	64	77	90	103	116	129	193
15.8	19	25.3	31.6	37.9	44.2	51	57	63	95
110	131	175	219	263	307	350	394	438	657
40.2	48.2	64	80	96	112	129	145	161	241
19.7	23.7	31.6	39.5	47.4	55	63	71	79	118
48.2	58	77	96	116	135	154	173	193	289
23.7	28.4	37.9	47.4	57	66	76	85	95	142
9.9	11.9	15.9	19.9	23.9	27.9	31.8	35.8	39.8	60
27.6	33.2	44.2	55	66	77	88	100	111	166
11.6	13.9	18.6	23.2	27.8	32.5	37.1	41.7	46.4	70
3.9	4.7	6.3	7.8	9.4	11	12.5	14.1	15.6	23.5
31.6	37.9	51	63	76	88	101	114	126	190
13.3	15.9	21.2	26.5	31.8	37.1	42.4	47.7	53	80
4.5	5.4	7.2	8.9	10.7	12.5	14.3	16.1	17.9	26.8
39.5	47.4	63	79	95	111	127	142	158	237
16.6	19.9	26.5	33.1	39.8	46.4	53	60	66	99
5.6	6.7	8.9	11.2	13.4	15.6	17.9	20.1	22.3	33.5

← LAMINAR FLOW →

For a liquid having a specific gravity other than 1.00, multiply the value from the table by the specific gravity. No allowance for aging of pipe is included.

TABLE 3 Continued.

gpm	Pipe size	Viscosity, SSU							
		100	200	300	400	500	1000	1500	2000
120	3	2.7	3.1	3.2	3.2	4	8	11.9	15.9
	4	0.73	0.81	0.81	1.1	1.3	2.7	4	5.4
	6	0.98	0.11	0.16	0.21	0.26	0.52	0.78	1.0
140	3	3.4	4	4.3	4.3	4.6	9.3	13.9	18.6
	4	0.95	1.1	1.1	1.3	1.6	3.1	4.7	6.3
	6	0.17	0.18	0.21	0.28	0.35	0.69	1.0	1.4
160	3	4.4	5	5.7	5.7	5.7	10.6	15.9	21.2
	4	1.2	1.4	1.4	1.4	1.8	3.6	5.4	7.2
	6	0.17	0.18	0.21	0.28	0.35	0.69	1.0	1.4
180	3	5.3	6.3	7	7	7	11.9	17.9	23.9
	4	1.5	1.8	1.8	1.8	2	4	6	8
	6	0.2	0.24	0.24	0.31	0.39	0.78	1.2	1.6
200	3	6.5	7.7	8.8	8.8	8.8	13.3	19.9	26.5
	4	1.8	2.2	2.2	2.2	2.2	4.5	6.7	8.9
	6	0.25	0.3	0.3	0.35	0.43	0.87	1.3	1.7
250	4	2.6	3.2	3.5	3.5	3.5	5.6	8.4	11.2
	6	0.36	0.43	0.45	0.45	0.51	1.1	1.6	2.2
	8	0.95	0.12	0.12	0.15	0.18	0.36	0.54	0.72
300	4	3.7	4.3	5	5	5	6.7	10.1	13.4
	6	0.5	0.6	0.65	0.65	0.65	1.3	2	2.6
	8	0.13	0.17	0.17	0.18	0.22	0.43	0.65	0.87
400	6	0.82	1	1.1	1.2	1.2	1.7	2.6	3.5
	8	0.23	0.27	0.29	0.29	0.29	0.58	0.87	1.2
	10	0.08	0.09	0.1	0.1	0.12	0.23	0.35	0.47
500	6	1.2	1.5	1.6	1.8	1.8	2.2	3.2	4.3
	8	0.33	0.39	0.44	0.47	0.47	0.72	1.1	1.5
	10	0.11	0.14	0.15	0.15	0.15	0.29	0.44	0.58
600	6	1.8	2.2	2.3	2.4	2.6	2.7	3.9	5.2
	8	0.47	0.57	0.62	0.67	0.67	0.87	1.3	1.7
	10	0.16	0.18	0.2	0.22	0.22	0.35	0.52	0.07
700	6	2.3	2.7	3	3.2	3.5	3.6	4.6	6.1
	8	0.6	0.74	0.82	0.89	0.93	1	1.5	2
	10	0.2	0.25	0.27	0.3	0.3	0.41	0.61	0.82
800	6	2.8	3.5	3.7	4	4.2	4.8	5.2	6.9
	8	0.78	0.94	1	1.1	1.2	1.2	1.7	2.3
	10	0.26	0.3	0.34	0.38	0.4	0.47	0.7	0.92
900	6	3.5	4.3	4.6	5.0	5.2	6	6	7.8
	8	0.95	1.1	1.3	1.4	1.5	1.5	2	2.6
	10	0.32	0.37	0.43	0.46	0.5	0.52	0.79	1.1
1000	8	1.1	1.4	1.5	1.6	1.8	1.9	2.2	2.9
	10	0.38	0.45	0.5	0.55	0.6	0.6	0.87	1.2
	12	0.17	0.2	0.22	0.24	0.25	0.29	0.43	0.58

← TURBULENT FLOW → ← LAMINAR FLOW →

TABLE 3 Continued.

Viscosity, SSU									
2500	3000	4000	5000	6000	7000	8000	9000	10,000	15,000
19.9	23.9	31.8	39.8	47.7	56	61	72	80	119
6.7	8	10.7	13.4	16.1	18.8	21.4	24.1	26.8	40.2
1.3	1.6	2.1	2.6	3.1	3.6	1.2	4.7	5.2	7.8
23.2	27.8	37.1	46.4	56	65	74	84	93	139
7.8	9.4	12.5	15.6	18.8	21.9	25	28.2	31.3	46.9
1.5	1.8	2.4	3.0	3.6	4.2	4.9	5.5	6.0	9.1
26.5	31.8	42.4	53	64	74	85	95	106	159
8.9	10.7	14.3	17.9	21.5	25	28.6	32.2	35.7	54
1.7	2.1	2.8	3.5	4.2	4.9	5.5	6.2	6.9	10.4
29.8	35.8	47.7	60	72	84	95	107	119	179
10.1	12.1	16.1	20.1	24.1	28.1	32.2	36.2	40.2	60
2	2.3	3.1	3.9	4.7	5.5	6.2	7	7.8	11.7
33.1	39.8	53	66	80	93	106	119	133	199
11.2	13.4	17.9	22.3	26.8	31.3	35.7	40.2	44.7	67
2.2	2.6	3.5	4.3	5.2	6.1	6.9	7.8	8.7	13
14	16.8	22.3	27.9	33.5	39.1	44.7	50	56	84
2.7	3.3	4.3	5.4	6.5	7.6	8.7	9.8	10.8	16.3
0.9	1.1	1.5	1.8	2.2	2.5	2.9	3.3	3.6	5.4
16.8	20.1	26.8	33.5	40.2	47	54	60	67	101
3.3	3.9	5.2	6.5	7.8	9.1	10.4	11.7	13	19.5
1.0	1.3	1.7	2.2	2.6	3	3.5	3.9	4.3	6.5
4.3	5.2	6.9	8.7	10.4	12.1	13.9	15.6	17.3	26
1.5	1.7	2.3	2.9	3.5	4.1	4.6	5.2	5.8	8.7
0.58	0.7	0.93	1.2	1.4	1.6	1.9	2.1	2.3	3.5
5.4	6.5	8.7	10.8	13	15.2	17.3	19.5	21.7	32.5
1.8	2.2	2.9	3.6	4.3	5.1	5.8	6.5	7.2	10.8
0.73	0.87	1.2	1.5	1.8	2	2.3	2.6	2.9	4.4
6.5	7.8	10.4	13	16	18.2	20.8	23.4	26	39
2.2	2.6	3.5	4.3	5.2	6.1	6.9	7.8	8.7	13
0.87	1.1	1.4	1.8	2.1	2.4	2.8	3.3	3.5	5.2
7.6	9.1	12.1	15.2	18.4	21.2	24.3	27.3	30.3	45.5
2.5	3	4.1	5.1	6.1	7.1	8.1	9.1	10.1	15.2
1	1.2	1.6	2	2.4	2.9	3.3	3.7	4.1	6.1
8.7	10.4	13.9	17.3	20.8	24.3	27.7	31.2	34.7	52
2.9	3.5	4.6	5.8	6.9	8.1	9.3	10.4	11.6	17.3
1.2	1.4	1.9	2.3	2.8	3.3	3.7	4.2	4.7	7
9.8	11.7	15.6	19.5	23.4	27.3	31.2	35.1	39	58.5
3.3	3.9	5.2	6.5	7.8	9.1	10.4	11.7	13	19.5
1.3	1.6	2.1	2.6	3.1	3.7	4.2	4.7	5.2	7.9
3.6	4.3	5.8	7.2	8.7	10.1	11.6	13	14.5	21.7
1.5	1.8	2.3	2.9	3.5	4.1	4.7	5.2	5.8	8.7
0.72	0.87	1.2	1.5	1.7	2	2.3	2.6	2.9	4.3


 LAMINAR FLOW

TABLE 3 Continued.

gpm	Pipe size	Viscosity, SSU					
		20,000	25,000	30,000	40,000	50,000	60,000
3	2	19.3	24.1	28.9	38.5	48.2	58
	2½	9.5	11.8	14.2	19	23.7	28.4
	3	4	5	6	8	9.9	11.9
5	2	32	40	48.2	64	80	96
	2½	15.8	19.7	23.7	31.6	39.5	47.4
	3	6.6	8.3	9.9	13.3	16.6	9.9
7	2	45	56	67	90	112	135
	2½	22.1	27.6	33.2	44.2	55	66
	3	9.3	10.6	13.9	18.6	23.2	27.8
10	2½	31.6	39.5	47.4	63	79	95
	3	13.3	16.6	19.9	26.5	33.1	39.8
	4	4.5	5.6	6.7	8.9	11.2	13.4
15	2½	47.4	59	71	95	118	142
	3	19.9	24.9	29.8	39.8	49.7	60
	4	6.7	8.4	10.1	13.4	16.8	20.1
20	3	26.5	33.1	39.8	53	66	80
	4	8.9	11.2	13.4	17.9	22.3	26.8
	6	1.7	2.2	2.6	3.5	4.3	5.2
25	3	33.1	41.4	49.7	66	83	99
	4	11.2	14	16.8	22.3	27.9	33.5
	6	2.2	2.7	3.3	4.3	5.4	6.5
30	3	39.8	49.7	60	80	99	119
	4	13.4	16.8	20.1	26.8	33.5	40.2
	6	2.6	3.3	3.9	5.2	6.5	7.8
40	3	53	66	80	106	133	160
	4	17.9	22.3	26.8	35.7	44.7	54
	6	3.5	4.3	5.2	6.9	8.7	10.4
50	4	22.3	27.9	33.5	44.7	56	67
	6	4.3	5.4	6.5	8.7	10.8	13
	8	1.5	1.8	2.7	2.9	3.6	4.3
60	4	26.8	33.5	40.2	54	67	80
	6	5.2	6.5	7.8	10.4	13	16
	8	1.7	2.2	2.6	3.5	4.3	5.2
70	4	31.3	39.1	46.9	63	78	94
	6	6.1	7.6	9.1	12.1	15.2	18.4
	8	2	2.5	3	4.1	5.1	6.1
80	6	6.9	8.7	10.4	13.9	17.3	20.8
	8	2.3	2.9	3.5	4.6	5.8	6.9
	10	0.93	1.2	1.4	1.9	2.3	2.8
90	6	7.8	9.8	11.7	15.6	19.5	23.4
	8	2.6	3.3	3.9	5.2	6.5	7.8
	10	1.1	1.3	1.6	2.1	2.6	3.1

← LAMINAR FLOW →

TABLE 3 Continued.

Viscosity, SSU								
70,000	80,000	90,000	100,000	125,000	150,000	175,000	200,000	500,000
67	77	87	96	120	145	169	193	482
332	37.9	42.6	47.4	59	71	83	95	237
13.9	15.9	17.9	19.9	24.9	29.8	34.8	39.8	99
112	129	145	161	200	241	281	321	803
55	63	71	79	99	118	138	158	395
23.2	26.5	29.8	33	41.4	49.7	58	66	166
157	180	202	225	281	337	393	450	...
77	88	100	111	138	166	194	221	553
32.5	37.1	40.7	46.4	58	70	81	93	232
111	126	142	158	197	237	276	316	790
46.4	53	60	66	83	99	116	133	331
15.6	17.9	20.1	22.3	27.9	33.5	39.1	44.7	112
166	190	213	237	296	355	415	474	...
70	80	89	99	124	149	174	199	497
23.5	26.8	30.2	33.5	41.9	50	59	67	168
93	106	119	133	166	199	232	265	663
31.3	35.7	40.2	44.7	56	67	78	89	223
6.1	6.9	7.8	8.7	10.8	13	15.2	17.3	43.3
116	133	149	166	207	49	290	331	828
39.1	44.7	50	56	70	84	98	112	279
7.6	8.7	9.8	10.8	13.5	16.3	19	21.7	54
139	159	179	199	249	298	348	398	...
46.9	54	60	67	84	101	117	134	335
9.1	10.4	11.7	13	16.3	19.5	22.7	26	65
186	212	239	265	331	398	464	532	...
63	72	80	89	112	134	156	179	447
12.1	13.9	15.6	17.3	21.7	26	30.3	34.7	87
78	89	101	112	140	168	196	223	559
15.2	17.3	19.5	21.7	27.1	32.5	37.9	43.3	107
5.1	5.8	6.5	7.2	9	10.8	12.6	14.5	36.1
94	107	121	134	168	201	235	268	670
18.2	20.8	23.4	26	32.5	39	45.5	52	130
6.1	6.9	7.8	8.7	10.8	13	15.2	17.3	43.4
110	125	141	156	196	235	274	313	782
21.2	24.3	27.3	30.3	37.9	45.5	53	61	152
7.1	8.1	9.1	10.1	12.6	15.2	17.7	20.2	51
24.3	27.7	31.2	34.7	43.3	52	61	69	173
8.1	9.3	10.4	11.6	14.5	17.3	20.2	23.1	58
3.3	3.7	4.2	4.7	5.8	7	8.2	9.3	23.3
27.3	31.2	35.1	39	48.7	59	68	78	195
9.1	10.4	11.7	13	16.3	19.5	22.8	26	65
3.7	4.2	4.7	5.2	6.6	7.9	9.2	10.5	26.2

← LAMINAR FLOW →

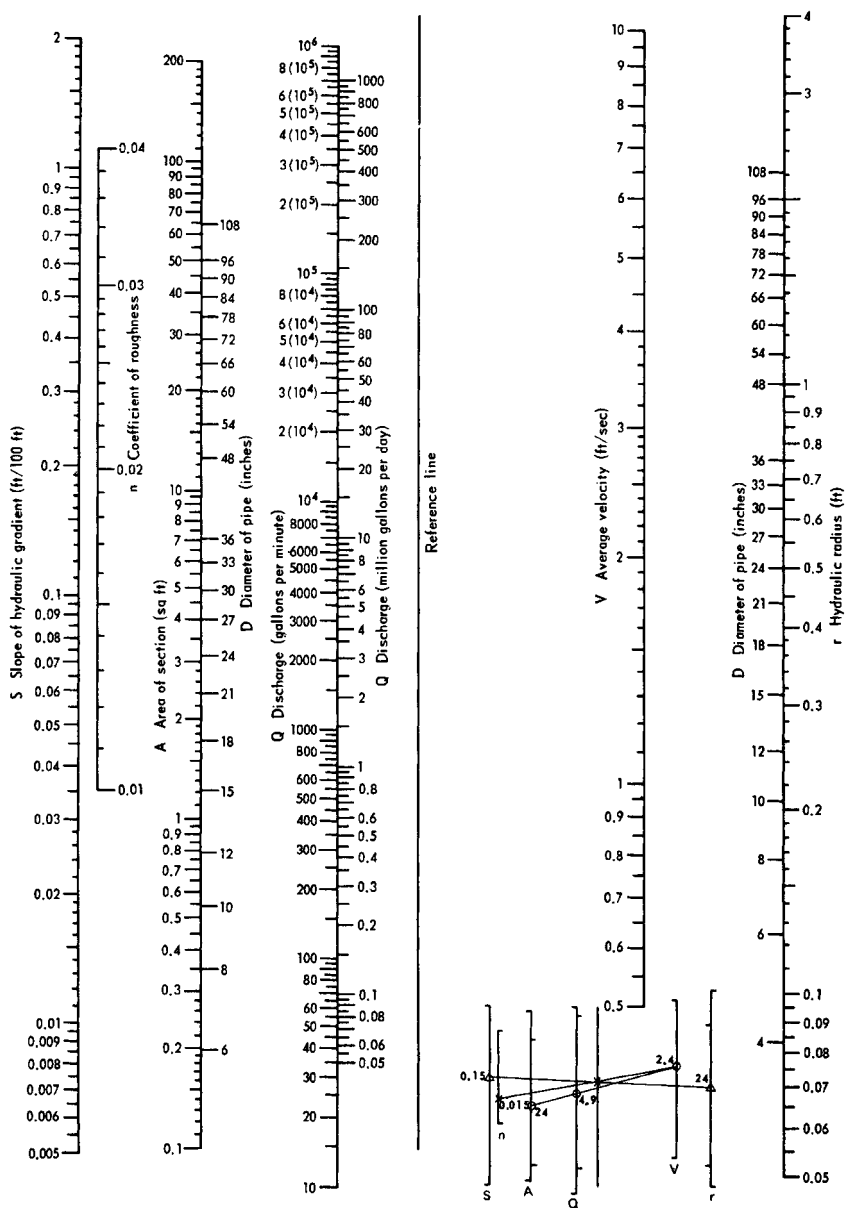
TABLE 3 Continued.

gpm	Pipe size	Viscosity, SSU					
		20,000	25,000	30,000	40,000	50,000	60,000
100	6	8.7	10.8	13	17.3	21.7	26
	8	2.9	3.6	4.3	5.8	7.2	8.7
	10	1.2	1.5	1.8	2.3	2.9	3.5
120	6	10.4	13	15.6	20.8	26	31.2
	8	3.5	4.3	5.2	6.9	8.7	10.4
	10	1.4	1.8	2.1	2.8	3.5	4.2
140	6	12.1	15.2	18.2	24.3	30.3	36.4
	8	4	5.1	6.1	8.1	10.1	12.1
	10	1.7	2	2.4	3.3	4.1	4.9
160	6	13.9	17.3	20.8	27.7	34.7	41.6
	8	4.6	5.8	6.9	9.3	11.6	13.8
	10	1.9	2.3	2.8	3.7	4.7	5.6
180	6	15.6	19.5	23.4	31.2	39	46.8
	8	5.2	6.5	7.8	10.4	13	15.6
	10	2.1	2.6	3.1	4.2	5.2	6.3
200	8	5.8	7.2	8.7	11.6	14.5	17.3
	10	2.3	2.9	3.5	4.7	5.8	7
	12	1.2	1.5	1.7	2.3	2.9	3.5
250	8	7.2	9	10.8	14.5	18.1	21.7
	10	2.9	3.6	4.4	5.8	7.3	8.7
	12	1.5	1.8	2.2	2.9	3.6	4.3
300	8	8.7	10.8	13	17.3	21.7	26
	10	3.5	4.4	5.2	7	8.7	10.5
	12	1.7	2.2	2.6	3.5	4.3	5.2
400	8	11.6	14.5	17.3	23	28.9	34.7
	10	4.7	5.8	7	9.3	11.6	14
	12	2.3	2.9	3.5	4.6	5.8	7
500	8	14.5	18.1	21.7	28.9	36.1	43.4
	10	5.8	7.3	8.7	11.6	14.6	17.5
	12	2.9	3.6	4.3	5.8	7.2	8.7
600	8	17.3	21.7	26	34.7	43.4	52
	10	7	8.7	10.5	14	17.5	21
	12	3.5	4.3	5.2	7	8.7	10.4
700	8	20.2	25.3	30.3	40.5	51	61
	10	8.2	10.2	12.2	16.3	20.4	24.4
	12	4.1	5.1	6.1	8.1	10.1	12.2
800	8	23.1	28.9	34.7	46.2	58	69
	10	9.3	11.6	14	18.6	23.3	27.9
	12	4.6	5.8	7	9.3	11.6	13.9
900	8	26	32.5	39	52	65	78
	10	10.5	13.1	15.7	21	26.2	31.4
	12	5.2	6.5	7.8	10.4	13	15.6
1000	8	28.9	36.1	43.4	58	72	87
	10	11.6	14.6	17.5	23.3	29.1	34.9
	12	5.8	7.2	8.7	11.6	14.5	17.4

**TABLE 3** Continued.

Viscosity, SSU								
70,000	80,000	90,000	100,000	125,000	150,000	175,000	200,000	500,000
30.3	34.7	39	43.3	54	65	76	87	217
10.1	11.6	13	14.5	18.1	21.7	25.3	28.9	72
4.2	4.7	5.2	5.8	7.3	8.7	10.2	11.6	29.1
36.4	41.6	46.8	52	65	78	91	104	260
12.1	13.9	15.6	17.3	21.7	26	30.4	34.7	87
4.9	5.6	6.3	7	8.7	10.5	12.2	14	34.9
42.5	48.5	55	61	76	91	106	121	303
14.2	16.2	18.2	20.2	25.3	30.4	35.4	40.5	101
5.7	6.5	7.3	8.1	10.2	12.2	14.3	16.3	40.7
48.5	56	62	69	87	104	121	139	347
16.2	18.5	20.8	23.1	28.9	34.7	40.5	46.2	116
6.5	7.5	8.4	9.3	11.6	14	16.3	18.6	46.6
55	62	70	78	98	117	137	156	390
18.2	20.8	23.4	26	32.5	39	45.5	52	130
7.3	8.4	9.4	10.5	13.1	15.7	18.3	21	52
20.2	23.1	26	28.9	36.1	43.4	51	58	145
8.2	9.3	10.5	11.6	14.6	17.5	20.4	23.3	58
4.1	4.6	5.2	5.8	7.2	8.7	10.1	11.6	28.9
25.3	28.9	32.5	36.1	45.2	54	63	72	181
10.2	11.6	13.1	14.6	18.2	21.8	25.5	29.1	73
5.1	5.8	6.5	7.2	9	10.9	12.7	14.5	36.2
30.4	34.7	39	43.4	54	65	76	87	217
12.2	14	15.7	17.5	21.8	26.2	30.6	34.9	87
6.1	7	7.8	8.7	10.9	13	15.2	17.4	43.4
40.5	46.2	52	58	72	87	101	116	289
16.3	18.6	21	23.3	29.6	34.9	40.7	46.6	116
8.1	9.3	10.4	11.6	14.5	17.4	20.3	23.2	58
51	58	65	72	90	108	126	145	361
20.4	23.3	26.2	29.1	36.4	43.7	51	58	146
10.1	11.6	13	14.5	18.1	21.7	25.3	28.9	72
61	69	78	87	107	130	152	173	434
24.4	27.9	31.4	34.9	43.7	52	61	70	175
12.2	13.9	15.6	17.4	21.7	26.1	30.4	34.7	87
71	81	91	101	126	152	177	202	506
28.5	32.6	36.7	40.7	51	61	71	82	204
14.2	16.2	18.2	20.3	25.3	30.4	35.5	40.5	101
81	93	104	116	145	173	202	231	578
32.6	37.3	41.9	46.6	58	70	82	93	233
16.2	18.5	20.8	23.1	28.9	34.7	40.5	46.3	116
91	104	117	130	163	195	228	260	650
36.7	41.9	47.1	52	66	79	92	105	262
18.2	20.8	23.4	26.1	32.6	39.1	45.6	52	130
101	116	130	145	181	217	253	289	723
40.7	46.6	52	58	72	87	102	116	291
20.3	23.2	26.1	28.9	36.2	43.4	51	58	145





**FIGURE 35** Nonogram for the solution of the Manning formula. Obtain values of  $n$  from Table 4. To solve for  $S$ , align  $A$  with  $Q$  and read  $V$ , align  $V$  and  $n$  intersecting reference line, align  $r$  with reference line intersection, and read  $S$  (1 m = 3.28 ft; 1 m = 39.37 in; 1 m<sup>2</sup> = 1550 in<sup>2</sup>; 1 m<sup>3</sup>/h = 4.4 gpm; 1 m/s = 3.28 ft/s; 1 m/m = 1 ft/ft).

**TABLE 4** Values of friction factor  $n$  to be used with the Manning Formula in Figure 35

Surface	Rest	Good	Fair	Bad
Uncoated cast iron pipe	0.012	0.013	0.014	0.015
Coated cast iron pipe	0.011	0.012 <sup>a</sup>	0.013 <sup>a</sup>	
Commercial wrought iron pipe, black	0.012	0.013	0.014	0.015
Commercial wrought iron pipe, galvanized	0.013	0.014	0.015	0.017
Smooth brass and glass pipe	0.009	0.010	0.011	0.013
Smooth lockbar and welded OD pipe	0.010	0.011 <sup>a</sup>	0.013 <sup>a</sup>	
Riveted and spiral steel pipe	0.013	0.015 <sup>a</sup>	0.017 <sup>a</sup>	
Vitrified sewer pipe	0.010	0.013 <sup>a</sup>	0.015	0.017
	0.011			
Common clay drainage tile	0.011	0.012 <sup>a</sup>	0.014 <sup>a</sup>	0.017
Glazed brickwork	0.011	0.012	0.013 <sup>a</sup>	0.015
Brick in cement mortar; brick sewers	0.012	0.013	0.015 <sup>a</sup>	0.017
Neat cement surfaces	0.010	0.011	0.012	0.013
Cement mortar surfaces	0.011	0.012	0.013 <sup>a</sup>	0.015
Concrete pipe	0.012	0.013	0.015 <sup>a</sup>	0.016
Wood-stave pipe	0.010	0.011	0.012	0.013
Plank flumes:				
Planed	0.010	0.012 <sup>a</sup>	0.013	0.014
Unplaned	0.011	0.013 <sup>a</sup>	0.014	0.015
With battens	0.012	0.015 <sup>a</sup>	0.016	
Concrete-lined channels	0.012	0.014 <sup>a</sup>	0.016 <sup>a</sup>	0.018
Cement-rubble surface	0.017	0.020	0.025	0.030
Dry-rubble surface	0.025	0.030	0.033	0.035
Dressed-ashlar surface	0.013	0.014	0.015	0.017
Semicircular metal flumes, smooth	0.011	0.012	0.013	0.015
Semicircular metal flumes, corrugated	0.0225	0.025	0.0275	0.030
Canals and ditches:				
Earth, straight and uniform	0.017	0.020	0.0225 <sup>a</sup>	0.025
Rock cuts, smooth and uniform	0.025	0.030	0.033 <sup>a</sup>	0.035
Rock cuts, jagged and irregular	0.035	0.040	0.045	
Winding sluggish canals	0.0225	0.025 <sup>a</sup>	0.0275	0.030
Dredged earth channels	0.025	0.0275 <sup>a</sup>	0.030	0.033
Canals with rough stony beds, weeds on earth banks	0.025	0.030	0.035 <sup>a</sup>	0.040
Earth bottom, rubble sides	0.028	0.030 <sup>a</sup>	0.033 <sup>a</sup>	0.035
Natural stream channels:				
(1) Clean, straight bank, full stage, no rifts or deep pools	0.025	0.0275	0.030	0.033
(2) Same as (1), but some weeds and stones	0.030	0.033	0.035	0.040
(3) Winding, some pools and shoals, clean	0.033	0.035	0.040	0.045
(4) Same as (3), lower stages, more ineffective slope and sections	0.040	0.045	0.050	0.055
(5) Same as (3), some 2, eeds and stones	0.035	0.040	0.045	0.050
(6) Same as (4), stony sections	0.045	0.050	0.055	0.060
(7) Sluggish river reaches, rather weedy or with very deep pools	0.050	0.060	0.070	0.080
(8) Very weedy reaches	0.075	0.100	0.125	0.150

<sup>a</sup>Values commonly used in designing.

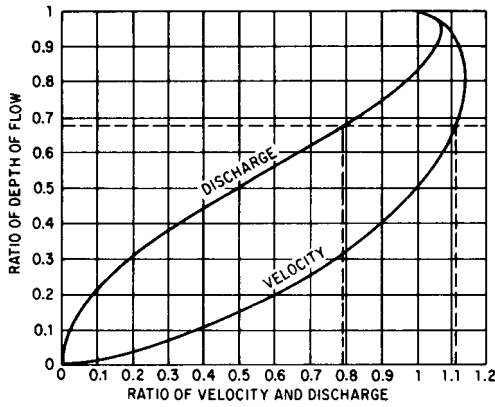


FIGURE 36 Discharge velocity of a partially full circular pipe versus that of a full pipe

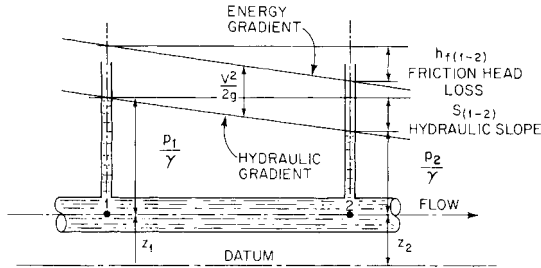


FIGURE 37 Slopes of energy and hydraulic gradients measure frictional head loss ft/ft (m/m) of pipe length

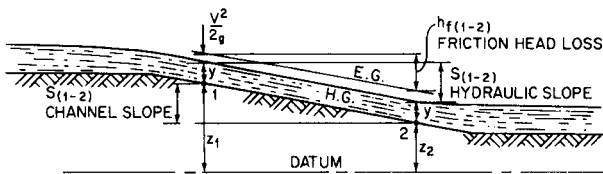


FIGURE 38 In an open channel with uniform flow, slopes of channel bottom, energy, and hydraulic gradients are the same as frictional head loss ft/ft (m/m) of channel length.

A rearrangement of Eq. 18 gives

in USCS units 
$$S = h_f = \left( \frac{Vn}{1.486r^{2/3}} \right)^2 \quad (19a)$$

in SI units 
$$S = h_f = \left( \frac{Vn}{r^{2/3}} \right)^2 \quad (19b)$$

The following examples illustrate the application of the Manning formula (Eq. 19), Figures 35 and 36, and Table 4 to the solution of problems involving the flow of water in open channels.

**EXAMPLE 10** The flow through a 24-in (61.0-cm) ID commercial wrought iron pipe in fair condition is 4.9 mgd (772.7 m<sup>3</sup>/h). Determine the loss of head as a result of friction in feet per 100 ft (meters per 100 m) of pipe and the slope required to maintain a full, uniformly flowing pipe.

$$\text{In USCS units} \quad V = \frac{\text{gpm}}{(\text{pipe ID in inches})^2} \times 0.408 = \frac{4.9 \times 10^6 \times 0.408}{24 \times 60 \times 24^2} = 2.41 \text{ ft/s}$$

$$\text{In SI units} \quad V = \frac{\text{m}^3/\text{h}}{(\text{pipe ID in cm})^2} \times 3.54 = \frac{772.7}{60.96^2} \times 3.54 = 0.736 \text{ m/s}$$

From Table 4,  $n = 0.015$ .

In USCS units

$$r = D/4 = 2/4 = 0.5 \text{ ft} \quad (\text{hydraulic radius})$$

$$S = h_f = \left( \frac{Vn}{1.486r^{2/3}} \right)^2 = \left( \frac{2.41 \times 0.015}{1.486 \times 0.5^{2/3}} \right)^2 \quad (\text{from Eq. 19a})$$

$$= 0.0015 \text{ ft/ft}$$

$$100 \times 0.0015 = 0.15 \text{ ft/100 ft} \quad (\text{slope and frictional head})$$

In SI units

$$r = D/4 = 0.6096/4 = 0.1524 \text{ m} \quad (\text{hydraulic radius})$$

$$S = h_f = \left( \frac{Vn}{r^{2/3}} \right)^2 = \left( \frac{0.736 \times 0.015}{0.1524^{2/3}} \right)^2 \quad (\text{from Eq. 19b})$$

$$= 0.0015 \text{ m/m}$$

$$100 \times 0.0015 = 0.15 \text{ m/100 m} \quad (\text{slope and frictional head})$$

The problem may also be solved by using Figure 35 and following the trace lines:

$$S = h_f = 0.15 \text{ ft/100 ft (m/100 m)}$$

**EXAMPLE 11** Determine what the flow and velocity would be if the pipe in Example 10 were flowing two-thirds full and were laid on the same slope.

Follow the trace lines in Figure 36 and note that the multipliers for discharge and velocity are 0.79 and 1.11, respectively. Therefore

$$\text{in USCS units} \quad \text{Flow} = 0.79 \times 4.9 = 3.87 \text{ mgd}$$

$$\text{Velocity} = 1.11 \times 2.41 = 2.68 \text{ ft/s}$$

$$\text{in SI units} \quad \text{Flow} = 0.79 \times 772.7 = 610 \text{ m}^3/\text{h}$$

$$\text{Velocity} = 1.11 \times 0.736 = 0.817 \text{ m/s}$$

**Pipe Fittings** Invariably, a system containing piping will have connections that change the size or direction of the conduit. These fittings add frictional losses, called *minor losses*, to the system head. Fitting losses are generally the result of changes in velocity or direction. A decreasing velocity results in more loss in head than an increasing velocity, as the former causes energy-dissipating eddies. Experimental results have

indicated that minor losses vary approximately as the square of the velocity through the fittings.

**VALVES AND STANDARD FITTINGS** The resistance to flow through valves and fittings may be found in References 4, 5, 17, and other sources. Losses are usually expressed in terms of a *resistance coefficient*  $K$  and the average velocity head in a pipe having the same diameter as the valve or fitting. The frictional resistance  $h$  in feet (meters) is found from the equation

$$h = K \frac{V^2}{2g} \quad (20)$$

where  $K$  = resistance coefficient, which depends on design and size of valve or fitting

$V$  = average velocity in pipe of corresponding internal diameter, ft/s (m/s)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

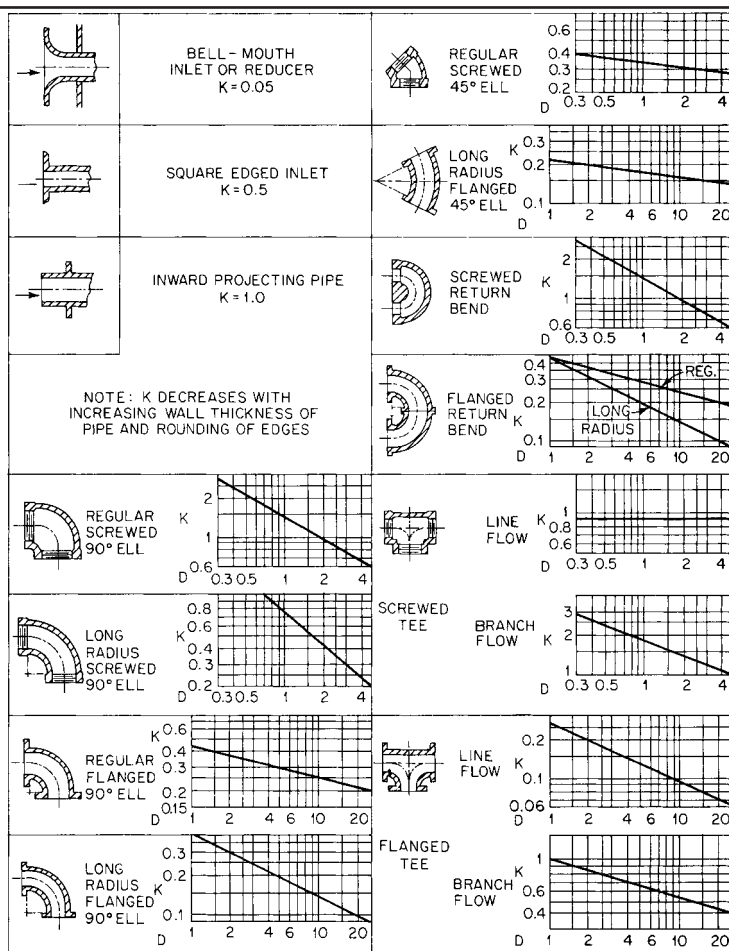
A comparison of the Darcy-Weisbach equation (Eq. 16) with Eq. 20 suggests that  $K$  equals  $f(L/D)$ , where  $L$  is the equivalent length of pipe in feet (meters) and  $D$  is the inside pipe diameter in feet (meters), to produce the same head loss in a straight pipe as through a valve or fitting. The friction in an "equivalent length of pipe" has been another method used to estimate head loss through valves and fittings. Values of the ratio  $L/D$  have been experimentally determined. This ratio multiplied by the inside diameter of a pipe of specified schedule for the valve or fitting being considered gives the equivalent length of pipe to use to calculate the head lost.

Loss of head in straight pipe depends on the friction factor or Reynolds number. However, with valves and fittings, head is lost primarily because of change in direction of flow, change in cross section, and obstructions in the flow path. For this reason, the resistance coefficient is practically constant for a particular shape of valve or fitting for all flow conditions, including laminar flow. The resistance coefficient would theoretically be constant for all sizes of a particular design of valve or fitting except that all sizes are not geometrically similar. The Crane Company has reported the results of tests that show that the resistance coefficient for a number of lines of valves and fittings decreases with increasing size at flow conditions of equal friction factor and that the equivalent length  $L/D$  tends to be constant for the various sizes at the same flow conditions.

When available, the  $K$  factor furnished by the valve or fitting manufacturer should always be used rather than the value from a general listing. The Hydraulic Institute lists losses in terms of  $K$  through valves and fittings (Tables 5a to 5c); these losses vary with size of the valve or fitting but are independent of friction factor. The Crane Company provides a similar listing of  $K$  values (Tables 6a to 6e). The latter listing of flow coefficients is associated with the velocity head  $V^2/2g$  that would occur through the internal diameter of the schedule pipes for the various ANSI classes of valves and fittings shown in Table 6e. If the connecting pipe is of a different size or schedule, either use the velocity for the pipe shown in Table 6a or use the actual pipe velocity head and correct the resistance coefficient obtained from this table by the multiplier

$$\left( \frac{\text{Actual pipe ID}}{\text{Standard pipe ID}} \right)^4$$

Tables 6 are based on the use of an equivalent length constant for complete turbulent flow for each valve or fitting shown. This constant is shown as the multiplier of the friction factor  $f_T$  for the corresponding clean commercial steel pipe with completely turbulent flow. The product  $(L/D)(f_T)$  is the coefficient  $K$ . The friction factors are given in Table 6a for different pipe sizes, or they can be obtained from Figure 31 or 32. If the valve or fitting has a sudden or gradual contraction, enlargement, or change in direction of flow, appropriate formulas for these conditions are given for the determination of  $K$ . If flow is laminar, valve and fitting resistance coefficients are obtained from Table 6a based on completely turbu-

**TABLE 5A** Resistance coefficients for  $K$  for valves and fittings

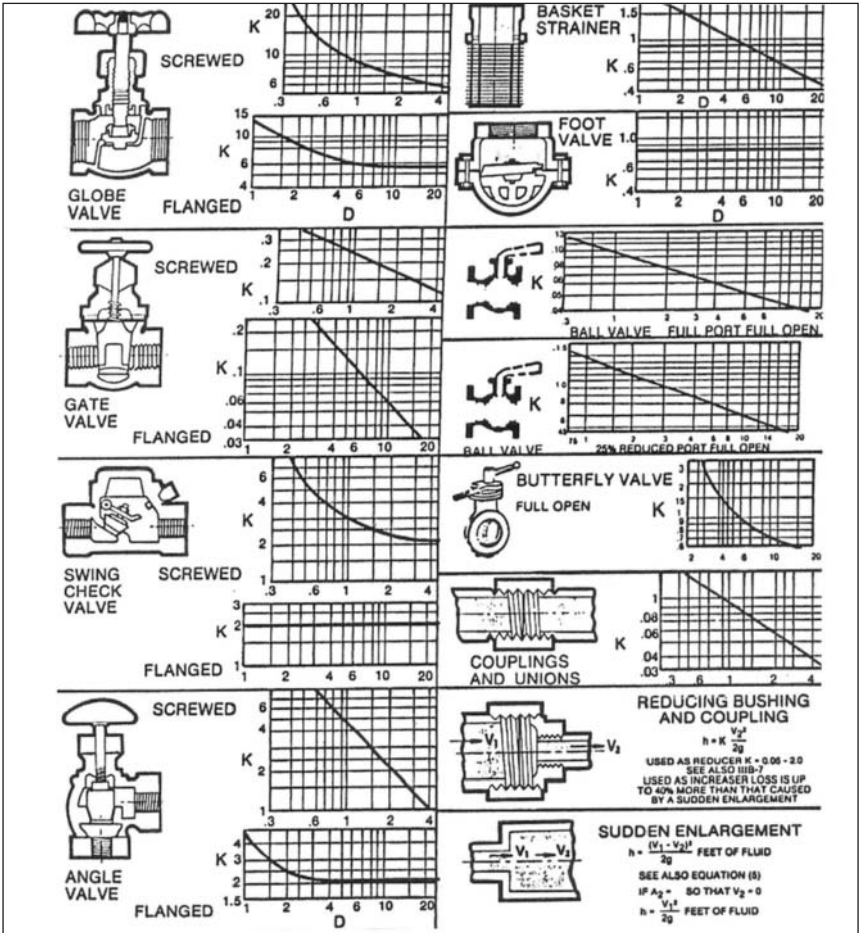
$$h = K \frac{V^2}{2g} \quad \text{FEET (METERS) OF FLUID}$$

NOTE:  $D$  = nominal iron pipe size in inches ( $\text{in} \times 25.4 = \text{mm}$ ). (Hydraulic Institute Engineering Data Book, Reference 5)

lent flow, but the pipe frictional loss is calculated using the laminar friction factor  $f = 64/Re$  instead of  $f_T$ . Also shown in Tables 6 for check valves is the minimum pipe velocity required for full disk lift for the coefficient of resistance listed ( $\bar{V}$  = liquid specific volume in cubic feet per pound).

Prior to the 15th printing (1976) of the Crane Company Technical Paper 410, and as shown in the first edition of this text, valve and fitting losses were calculated using an equivalent length of pipe rather than the coefficient  $K$ . The Crane Company states that this conceptual change regarding the values of equivalent length  $L/D$  and resistance coefficient  $K$  for valves and fittings relative to the friction factor in pipes has a relatively minor

**TABLE 5B** Resistance coefficients  $K$  for valves and fittings



NOTE:  $D$  = nominal iron pipe size in inches ( $\text{in} \times 25.4 = \text{mm}$ ). For velocities below 15 ft/s (4.6 m/s), check valves and foot valves will be only partially open and will exhibit higher values of  $K$  than shown.

$$h = K \frac{V^2}{2g}, \text{ ft (m) of fluid}$$

(Hydraulic Institute Engineering Data Book, Reference 5)

effect on most problems dealing with turbulent flow and avoids a significant overstatement of pressure drop in the laminar zone.

**Valve Flow Coefficient** The loss of head through valves, particularly control valves, is often expressed in terms of a *flow coefficient*  $C_v$  in USCS units ( $K_v$  in SI units). The flow of water in gallons per minute (cubic meters per hour) at 60°F (15.6°C) that will pass through a valve with a 1-lb/in<sup>2</sup> (1-bar) pressure drop is defined as the flow coefficient for a particular valve opening. Because loss of head  $h$  is a measure of energy loss per unit weight (force) and because  $h = p/\gamma$  and head loss varies directly with the square of the flow through a certain fixed opening, the formulas for flow coefficient are

**TABLE 5C** Approximate variation for  $K$  listed in Tables 5a and 5b

	Fitting	Range of variation, %
90° elbow	Regular screwed	±20 above 2-in size <sup>a</sup>
	Regular screwed	±40 below 2-in size
	Long radius, screwed	±25
	Regular flanged	±35
	Long radius, flanged	±30
45° elbow	Regular screwed	±10
	Long radius, flanged	±10
180° bend	Regular screwed	±25
	Regular flanged	±35
	Long radius, flanged	±30
T	Screwed, line or branch flow	±25
	Flanged, line or branch flow	±35
Globe valve	Screwed	±25
	Flanged	±25
Gate valve	Screwed	±25
	Flanged	±50
Check valve <sup>b</sup>	Screwed	±30
	Flanged	{ +200 -80
Sleeve check valve	—	Multiply flanged values by 0.2 to 0.5
Tilting check valve	—	Multiply flanged values by 0.13 to 0.19
Drainage gate check	—	Multiply flanged values by 0.03 to 0.07
Angle valve	Screwed	±20
	Flanged	±50
Basket strainer	—	±50
Foot valve <sup>a</sup>	—	±50
Couplings	—	±50
Unions	—	±50
Reducers	—	±50

<sup>a</sup>In 3 25.4 = mm.<sup>b</sup>For velocities below 15 ft/s (4.6 m/s), check valves and foot valves will be only partially open and will exhibit higher values of  $K$  than shown.

Source: Reference 9.

$$\text{In USCS units} \quad C_v = \text{gpm} \sqrt{\frac{\text{sp. gr.}}{\text{lb/in}^2}} \quad (21a)$$

$$\text{also} \quad C_v = 29.9d^2/\sqrt{K} \quad (22a)$$

$$\text{In SI units} \quad K_v = \text{m}^3/\text{h} \sqrt{\frac{\text{sp. gr.}}{\text{bar}}} \quad (21b)$$

$$\text{also} \quad K_v = 0.04d^2/\sqrt{K} \quad (22b)$$

where  $d$  = internal diameter of pipe corresponding to  $K$  and as shown in Table 6e, in (mm) and 1 bar = 100 kPa. The conversion from the SI flow coefficient to the USCS flow coefficient is



**TABLE 6A** Resistance coefficient  $K$  for valves and fittings

**PIPE FRICTION DATA FOR CLEAN COMMERCIAL STEEL PIPE  
WITH FLOW IN ZONE OF COMPLETE TURBULENCE**

Nominal Size	1/2"	3/4"	1"	1 1/4"	1 1/2"	2"	2 1/2, 3"	4"	5"	6"	8-10"	12-16"	18-24"
Friction Factor ( $f_f$ )	.027	.025	.023	.022	.021	.019	.018	.017	.016	.015	.014	.013	.012

**FORMULAS FOR CALCULATING "K" FACTORS  
FOR VALVES AND FITTINGS WITH REDUCED PORT**

• Formula 1

$$K_2 = \frac{0.8 \sin \frac{\theta}{2} (1 - \beta^2)}{\beta^4} = \frac{K_1}{\beta^4}$$

• Formula 2

$$K_2 = \frac{0.5 (1 - \beta^2) \sqrt{\sin \frac{\theta}{2}}}{\beta^4} = \frac{K_1}{\beta^4}$$

• Formula 3

$$K_2 = \frac{2.6 \sin \frac{\theta}{2} (1 - \beta^2)^2}{\beta^4} = \frac{K_1}{\beta^4}$$

• Formula 4

$$K_2 = \frac{(1 - \beta^2)^2}{\beta^4} = \frac{K_1}{\beta^4}$$

• Formula 5

$$K_2 = \frac{K_1}{\beta^4} + \text{Formula 1} + \text{Formula 3}$$

$$K_2 = \frac{K_1 + \sin \frac{\theta}{2} [0.8 (1 - \beta^2) + 2.6 (1 - \beta^2)^2]}{\beta^4}$$

• Formula 6

$$K_2 = \frac{K_1}{\beta^4} + \text{Formula 2} - \text{Formula 4}$$

$$K_2 = \frac{K_1 + 0.5 \sqrt{\sin \frac{\theta}{2}} (1 - \beta^2) + (1 - \beta^2)^2}{\beta^4}$$

• Formula 7

$$K_2 = \frac{K_1}{\beta^4} + \beta \quad (\text{Formula 2} + \text{Formula 4}) \quad \text{when } \theta = 180^\circ$$

$$K_2 = \frac{K_1 + \beta [0.5 (1 - \beta^2) + (1 - \beta^2)^2]}{\beta^4}$$

NOTES:

$$\beta = \frac{d_1}{d_2}$$

$$\beta^2 = \left( \frac{d_1}{d_2} \right)^2 = \frac{a_1}{a_2}$$

Subscript 1 defines dimensions and coefficients with reference to the smaller diameter.

Subscript 2 refers to the larger diameter.

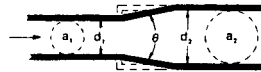
**SUDDEN AND GRADUAL CONTRACTION**



If:  $\theta \approx 45^\circ \dots \dots \dots K_2 = \text{Formula 1}$

$45^\circ < \theta < 180^\circ \dots K_2 = \text{Formula 2}$

**SUDDEN AND GRADUAL ENLARGEMENT**



If:  $\theta \approx 45^\circ \dots \dots \dots K_2 = \text{Formula 3}$

$45^\circ < \theta < 180^\circ \dots K_2 = \text{Formula 4}$

NOTE:  $K$  is based on use of schedule pipe as listed in Table 6e. In  $\times 25.4 = \text{mm}$ .

SOURCE: Reference 4.

$$C_v = 1.156K_v$$

**EXAMPLE 12** A pumping system consists of 20 ft (6.1 m) of 2-in (51-mm) suction pipe and 300 ft (91.5 m) of 1 1/2-in (38-mm) discharge pipe, both Schedule 40 new steel. Also included are a bell mouth inlet, a 90° short radius (SR) suction elbow, a full port suc-

TABLE 6B Resistance coefficient  $K$  for valves and fittings

## GATE VALVES

Wedge Disc, Double Disc, or Plug Type

If:  $\beta = 1, \theta = 0 \dots K_1 = 8 f_T$   
 $\beta < 1$  and  $\theta \approx 45^\circ \dots K_2 = \text{Formula 5}$   
 $\beta < 1$  and  $45^\circ < \theta \approx 180^\circ \dots K_2 = \text{Formula 6}$

## SWING CHECK VALVES

$K = 100 f_T$

Minimum pipe velocity (fps) for full disc lift  
 $= 35 \sqrt{V}$

$K = 50 f_T$

Minimum pipe velocity (fps) for full disc lift  
 $= 48 \sqrt{V}$

## GLOBE AND ANGLE VALVES

If:  $\beta = 1 \dots K_1 = 340 f_T$

If:  $\beta = 1 \dots K_1 = 55 f_T$

## LIFT CHECK VALVES

If:  $\beta = 1 \dots K_1 = 600 f_T$   
 $\beta < 1 \dots K_2 = \text{Formula 7}$

Minimum pipe velocity (fps) for full disc lift  
 $= 40 \beta^2 \sqrt{V}$

If:  $\beta = 1 \dots K_1 = 55 f_T$   
 $\beta < 1 \dots K_2 = \text{Formula 7}$

Minimum pipe velocity (fps) for full disc lift  
 $= 140 \beta^2 \sqrt{V}$

If:  $\beta = 1 \dots K_1 = 150 f_T$  If:  $\beta = 1 \dots K_1 = 55 f_T$

All globe and angle valves, whether reduced seat or throttled,

If:  $\beta < 1 \dots K_2 = \text{Formula 7}$

## TILTING DISC CHECK VALVES

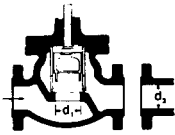
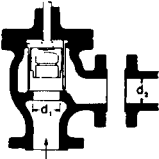

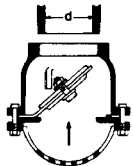
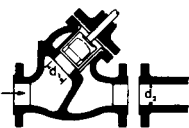
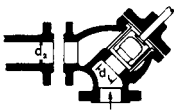
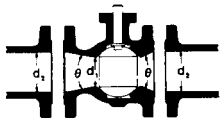

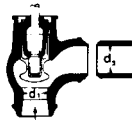
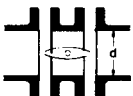
	$\alpha = 5^\circ$	$\alpha = 15^\circ$
Sizes 2 to 8" $\dots K =$	40 $f_T$	120 $f_T$
Sizes 10 to 14" $\dots K =$	30 $f_T$	90 $f_T$
Sizes 16 to 48" $\dots K =$	20 $f_T$	60 $f_T$
Minimum pipe velocity (fps) for full disc lift =	80 $\sqrt{V}$	30 $\sqrt{V}$

tion gate valve of Class 150 steel, a full port discharge gate valve of Class 400 steel, and a swing check valve. The valves and fittings are screw-connected and the same size as the connecting pipe.

Determine the pipe, valve, and fitting losses when 60°F (15.6°C) oil having a specific gravity of 0.855 is pumped at a rate of 60 gpm (13.6 m³/h). Use resistance coefficients from Tables 5.

The inner diameter of the suction pipe is 2.067 in (52.5mm), and from Figure 32.  $\epsilon/D = 0.00087$ . From Eq. 9,

**TABLE 6C** Resistance coefficient  $K$  for valves and fittings

<b>STOP-CHECK VALVES</b> (Globe and Angle Types)		<b>FOOT VALVES WITH STRAINER</b>	
 		<b>Poppet Disc</b> 	<b>Hinged Disc</b> 
<p>If: <math>\beta = 1 \dots K_1 = 400 f_T</math>  <math>\beta &lt; 1 \dots K_2 = \text{Formula 7}</math></p> <p>Minimum pipe velocity for full disc lift  <math>= 55 \beta^2 \sqrt{V}</math></p>		<p><math>K = 420 f_T</math></p> <p>Minimum pipe velocity (fps) for full disc lift  <math>= 15 \sqrt{V}</math></p>	<p><math>K = 75 f_T</math></p> <p>Minimum pipe velocity (fps) for full disc lift  <math>= 35 \sqrt{V}</math></p>
 		<b>BALL VALVES</b> 	
<p>If: <math>\beta = 1 \dots K_1 = 300 f_T</math>  <math>\beta &lt; 1 \dots K_2 = \text{Formula 7}</math></p> <p>Minimum pipe velocity (fps) for full disc lift  <math>= 60 \beta^2 \sqrt{V}</math></p>		<p>If: <math>\beta = 1, \theta = 0 \dots K_1 = 3 f_T</math>  <math>\beta &lt; 1 \text{ and } \theta \approx 45^\circ \dots K_2 = \text{Formula 5}</math>  <math>\beta &lt; 1 \text{ and } 45^\circ &lt; \theta \approx 180^\circ \dots K_2 = \text{Formula 6}</math></p>	
 		<b>BUTTERFLY VALVES</b> 	
<p>If: <math>\beta = 1 \dots K_1 = 55 f_T</math>  <math>\beta &lt; 1 \dots K_2 = \text{Formula 7}</math></p> <p>Minimum pipe velocity (fps) for full disc lift  <math>= 140 \beta^2 \sqrt{V}</math></p>		<p>Sizes 2 to 8" <math>\dots K = 45 f_T</math>          Sizes 10 to 14" <math>\dots K = 35 f_T</math>          Sizes 16 to 24" <math>\dots K = 25 f_T</math></p>	

In USCS units,  $V = \frac{\text{gpm}}{(\text{pipe ID in inches})^2} \times 0.408 = \frac{60}{2.067^2} \times 0.408 = 5.73 \text{ ft}$

$VD'' = 5.73 \times 2.067 = 11.8 \text{ ft/s} \times \text{in}$

In SI units  $V = \frac{\text{m}^3/\text{h}}{(\text{pipe ID in cm})^2} \times 3.54 = \frac{13.6}{5.25^2} \times 3.54 = 1.75 \text{ m/s}$

TABLE 6D Resistance coefficient  $K$  for valves and fittings

## PLUG VALVES AND COCKS

### Straight-Way

If:  $\beta = 1$ ,  
 $K_1 = 18 f_T$

### 3-Way

If:  $\beta = 1$ ,  
 $K_1 = 30 f_T$

If:  $\beta = 1$ ,  
 $K_1 = 90 f_T$

If:  $\beta < 1 \dots K_2 = \text{Formula 6}$

## MITRE BENDS

$\alpha$	$K$
0°	2 $f_T$
15°	4 $f_T$
30°	8 $f_T$
45°	15 $f_T$
60°	25 $f_T$
75°	40 $f_T$
90°	60 $f_T$

## 90° PIPE BENDS AND FLANGED OR BUTT-WELDING 90° ELBOWS

$r/d$	$K$	$r/d$	$K$
1	20 $f_T$	8	24 $f_T$
1.5	14 $f_T$	10	30 $f_T$
2	12 $f_T$	12	34 $f_T$
3	12 $f_T$	14	38 $f_T$
4	14 $f_T$	16	42 $f_T$
6	17 $f_T$	20	50 $f_T$

The resistance coefficient,  $K_B$ , for pipe bends other than 90° may be determined as follows:

$$K_B = (n - 1) \left( 0.25 \pi f_T \frac{r}{d} + 0.5 K \right) + K$$

$n$  = number of 90° bends

$K$  = resistance coefficient for one 90° bend (per table)

## CLOSE PATTERN RETURN BENDS

$K = 50 f_T$

## STANDARD ELBOWS

90°

$K = 30 f_T$

45°

$K = 16 f_T$

## STANDARD TEES

Flow thru run.....  $K = 20 f_T$

Flow thru branch.....  $K = 60 f_T$

## PIPE ENTRANCE

Inward  
Projecting

$K = 0.78$

Flush

For  $K$ ,  
see table

$r/d$	$K$
0.00°	0.5
0.02	0.28
0.04	0.24
0.06	0.15
0.10	0.09
0.15 & up	0.04

\*Sharp-edged

## PIPE EXIT

Projecting

$K = 1.0$

Sharp-Edged

$K = 1.0$

Rounded

$K = 1.0$

$$VD = 1.75 \times 0.0525 = 0.0919 \text{ m/s} \times \text{m}$$

$$(VD'' = 0.0919 \times 129.2 = 11.8 \text{ ft/s} \times \text{in})$$

From Figure 33,  $Re = 1 \times 10^4$ , and from Figure 31,  $f = 0.031$ . From Eq. 16

$$\text{in USCS units} \quad h_{fs} = 0.031 \frac{20 \times 12}{2.067} \times \frac{5.73^2}{2 \times 32.17} = 1.84 \text{ ft}$$

**TABLE 6E** Pipe schedule for different classes of valves and fittings associated with  $K$  factors used in Tables 6A to 6D

Glass	Schedule
300 and lower	40
400 and 600	80
900	120
1500	160
2500 (Sizes $\frac{1}{2}$ to 6 in) <sup>a</sup>	XXS
2500 (sizes 8 in and up) <sup>a</sup>	160

<sup>a</sup>In  $\times 25.4 = \text{mm}$ .  
Source: Reference 4.

In SI units 
$$h_{fs} = 0.031 \frac{6.1}{0.0525} \times \frac{1.75^2}{2 \times 9.807} = 0.56 \text{ m}$$

The inner diameter of the discharge pipe is 1.610 in (40.89 mm), and from Figure 32,  $\varepsilon/D = 0.0011$ . From Eq. 9

in USCS units 
$$V = \frac{60}{1.601^2} \times 0.408 = 9.44 \text{ ft/s}$$
$$VD'' = 9.44 \times 1.601 = 15.2 \text{ ft/s} \times \text{in}$$

in SI units 
$$V = \frac{13.6}{4.089^2} \times 3.54 = 2.88 \text{ m/s}$$
$$VD = 2.88 \times 0.04089 = 0.118 \text{ m/s} \times \text{m}$$
$$(VD'' = 0.118 \times 129.2 = 15.2 \text{ ft/s} \times \text{in})$$

From Figure 33,  $Re = 1.5 \times 10^4$ , and from Figure 31,  $f = 0.030$ . From Eq. 16

in USCS units 
$$h_{fd} = 0.030 \frac{300 \times 12}{1.610} \times \frac{9.44^2}{2 \times 32.17} = 9.29 \text{ ft}$$

in SI units 
$$h_{fd} = 0.030 \frac{91.5}{0.04089} \times \frac{2.88^2}{2 \times 9.807} = 28.39 \text{ m}$$

The valve and fitting losses from Tables 5 and Eq. 20 are 2-in (51-mm) bellmouth,  $K = 0.05$ :

In USCS units 
$$h_{f1} = 0.05 \frac{5.73^2}{2 \times 32.17} = 0.026 \text{ ft}$$

In SI units 
$$h_{f1} = 0.05 \frac{1.75^2}{2 \times 9.807} = 0.0078 \text{ m}$$

2-in (51-mm) SR 90° elbow,  $K = 0.95 \pm 30\%$

In USCS units 
$$h_{f2} = 0.95 \frac{5.73^2}{2 \times 32.17} = 0.48 \pm 0.14 \text{ ft}$$

In SI units 
$$h_{f2} = 0.95 \frac{1.75^2}{2 \times 9.807} = 0.15 \pm 0.044 \text{ m}$$

2-in (51-mm) gate valve,  $K = 0.16 \pm 25\%$

In USCS units 
$$h_{f3} = 0.16 \frac{5.73^2}{2 \times 32.17} = 0.082 \pm 0.021 \text{ ft}$$

In SI units  $h_{f3} = 0.16 \frac{1.75^2}{2 \times 9.807} = 0.0250 \pm 0.0062 \text{ m}$   
 1½-in (38-mm) gate valve,  $K = 0.19 \pm 25\%$

In USCS units  $h_{f4} = 0.19 \frac{9.44^2}{2 \times 32.17} = 2.63 \pm 0.066 \text{ ft}$

In SI units  $h_{f4} = 0.19 \frac{2.88^2}{2 \times 9.807} = 0.0803 \pm 0.02 \text{ m}$

1½-in (38-mm) swing check valve,  $K = 2.5 \pm 30\%$

In USCS units  $h_{f5} = 2.5 \frac{9.44^2}{2 \times 32.17} = 3.46 \pm 1.0 \text{ ft}$

In SI units  $h_{f5} = 2.5 \frac{2.88^2}{2 \times 9.807} = 1.06 \pm 0.32 \text{ m}$

The total pipe, valve, and fitting losses are

In USCS units  $\Sigma h_f = h_{fs} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5}$   
 $= 1.84 + 92.9 + 0.026 + 0.48 + 0.082 + 0.263$   
 $+ 3.46 = 99.05 \text{ ft}$

Total variation =  $\pm (0.14 + 0.021 + 0.066 + 1.0) = \pm 1.23 \text{ ft}$

In SI units  $\Sigma h_f = h_{fs} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5}$   
 $= 0.56 + 28.39 + 0.0078 + 0.044 + 0.0250 + 0.0803$   
 $+ 1.06 = 30.17 \text{ m}$

Total variation =  $\pm (0.044 + 0.0062 + 0.02 + 0.32) = \pm 0.39 \text{ m}$

EXAMPLE 13 Solve Example 12 using resistance coefficients from Tables 6.

Suction pipe:

In USCS units  $h_{fs} = 1.84 \text{ ft}$  (same as in Example 12)

In SI units  $h_{fs} = 0.56 \text{ m}$  (same as in Example 12)

Discharge pipe,

In USCS units  $h_{fd} = 92.9 \text{ ft}$  (same as in Example 12)

In SI units  $h_{fd} = 28.39 \text{ m}$  (same as in Example 12)

Valve and fitting losses from Tables 6 and Eq. 20: 2-in (51-mm) bellmouth,  $K = 0.04$

In USCS units  $h_{f1} = 0.04 \frac{5.73^2}{2 \times 32.17} = 0.020 \text{ ft}$

In SI units  $h_{f1} = 0.04 \frac{1.75^2}{2 \times 9.807} = 0.0062 \text{ m}$

2-in (51-mm) SR 90° elbow,  $K = 30 f_T$   
 $f_T = 0.019$  (from Table 6A)

$K = 30 \times 0.019 = 0.57$

In USCS units  $h_{f2} = 0.57 \frac{5.73^2}{2 \times 32.17} = 0.29 \text{ ft}$

In SI units 
$$h_{f2} = 0.57 \frac{1.57^2}{2 \times 9.807} = 0.089 \text{ m}$$

2-in (51-mm) gate valve,  $\beta = 1, \theta = 0, k = 8f_T \quad \left( \beta = \frac{d_1}{d_2} = 1 \text{ from Table 6A} \right)$   

$$K = 8 \times 0.019 = 0.15$$

In USCS units 
$$h_{f3} = 0.15 \frac{5.73^2}{2 \times 32.17} = 0.077 \text{ ft}$$

In SI units 
$$h_{f3} = 0.15 \frac{1.75^2}{2 \times 9.807} = 0.023 \text{ m}$$

1-in (38-mm) gate valve,  $\beta = 1, \theta = 0, K = 8f_T$  for Schedule 80 (from Table 6E)  

$$K = 8f_T (1.10/1.500)^4 = 10.62f_T \quad \text{for Schedule 40}$$
  

$$f_T = 0.021 \quad \text{(from Table 6A)}$$
  

$$K = 10.62 \times 0.021 = 0.22$$

In USCS units 
$$h_{f4} = 0.22 \frac{9.44^2}{2 \times 32.17} = 0.30 \text{ ft}$$

In SI units 
$$h_{f4} = 0.22 \frac{2.88^2}{2 \times 9.807} = 0.093 \text{ m}$$

$\frac{1}{2}$ -in (38-mm) swing check valve,  $K = 100f_T$  (from Table 6B)

Minimum pipe velocity for full disk lift  $= 35\sqrt{V} = 35\sqrt{0.0189} = 4.81 \text{ ft/s} < 9.44 \text{ ft/s}$   

$$K = 100 \times 0.021 = 2.1$$

In USCS units 
$$h_{f5} = 2.1 \frac{9.44^2}{2 \times 32.17} = 2.91 \text{ ft}$$

In SI units 
$$h_{f5} = 2.1 \frac{2.88^2}{2 \times 9.807} = 0.89 \text{ m}$$

Total pipe, valve, and fitting losses:

In USCS units 
$$\begin{aligned} \Sigma h_f &= h_{f8} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5} \\ &= 1.84 + 92.9 + 0.020 + 0.29 + 0.077 + 0.30 + 2.91 \\ &= 98.34 \text{ ft} \end{aligned}$$

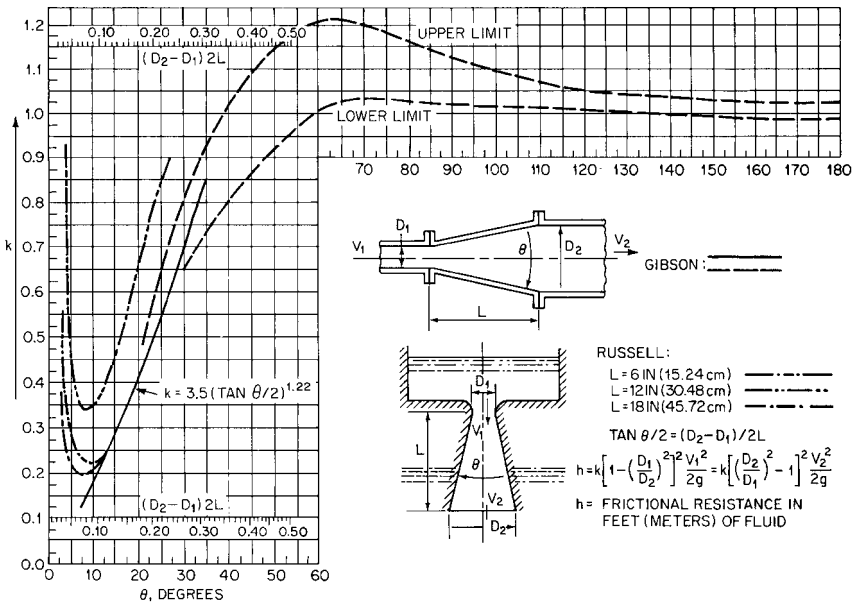
In SI units 
$$\begin{aligned} \Sigma h_f &= h_{f8} + h_{fd} + h_{f1} + h_{f2} + h_{f3} + h_{f4} + h_{f5} \\ &= 0.56 + 28.39 + 0.062 + 0.089 + 0.023 + 0.093 + 0.89 \\ &= 30.05 \text{ m} \end{aligned}$$

**INCREASERS** The head lost when there is a sudden increase in pipe diameter, with velocity changing from  $V_1$  to  $V_2$  in the direction of flow, can be calculated analytically. Computed results have been confirmed experimentally to be true to within  $\pm 3\%$ . The head loss is expressed as shown, with  $K$  computed to be equal to unity:

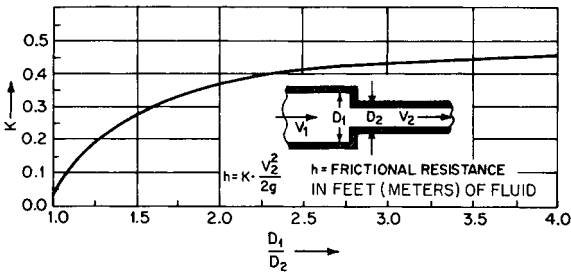
$$h = K \frac{(V_1 - V_2)^2}{2g} = K \left[ 1 - \left( \frac{D_1}{D_2} \right)^2 \right]^2 \frac{V_1^2}{2g} = K \left[ \left( \frac{D_2}{D_1} \right)^2 - 1 \right]^2 \frac{V_2^2}{2g} \quad (23)$$

The value of  $K$  is also approximately equal to unity if a pipe discharges into a relatively large reservoir. This indicates that all the kinetic energy  $V_1^2/2g$  is lost and  $V_2$  equals zero.

The loss of head for a gradual increase in pipe diameter when the flow is through a diffuser can be found from Figure 39 and Table 6A. The diffuser converts some of the kinetic



**FIGURE 39** Resistance coefficients for increasers and diffusers ( $D =$  in or ft [m];  $V =$  ft/s [m/s]) (Hydraulic Institute Engineering Data Book, Reference 5)



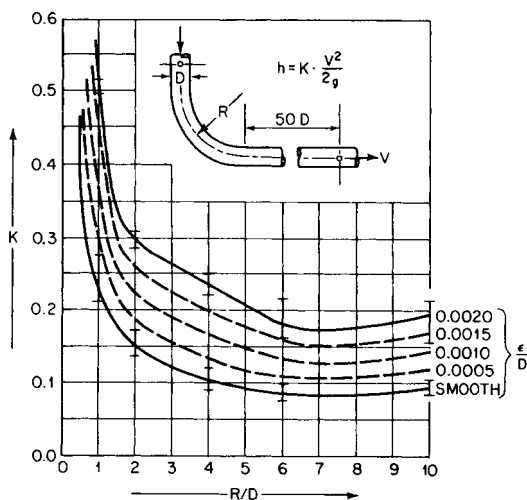
**FIGURE 40** Resistance coefficients for reducers (Hydraulic Institute Engineering Data Book, Reference 5)

energy to pressure. Values for the coefficient used with Eq. 23 for calculating head loss are shown in Figure 39. The optimum total angle appears to be  $7.5^\circ$ . Angles greater than this result in shorter diffusers and less friction, but separation and turbulence occur. For angles greater than  $50^\circ$ , it is preferable to use a sudden enlargement.

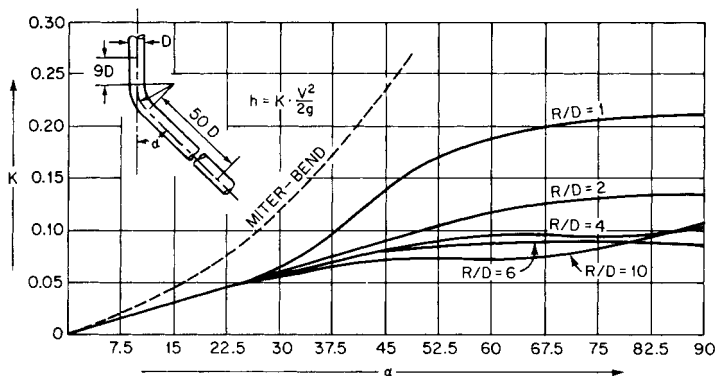
**REDUCERS** Figure 40 and Table 6A give values of the resistance coefficient to be used for sudden reducers.

**BENDS** Figure 41 may be used to determine the resistance coefficient for  $90^\circ$  pipe bends of uniform diameter. Figure 42 gives resistance coefficients for bends that are less than  $90^\circ$  and can be used for surfaces having moderate roughness such as clean steel and cast iron. Figures 41 and 42 are not recommended for elbows with  $R/D$  below 1. Tables 6D and 7 give values of resistance coefficients for miter bends.





**FIGURE 41** Resistance coefficients for 90° bends of uniform diameter (Hydraulic Institute Engineering Data Book, Reference 5)









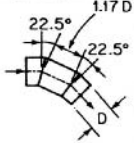
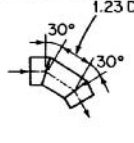
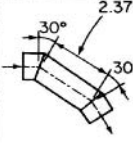
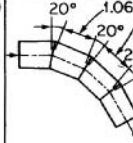
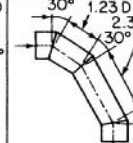
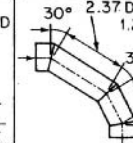
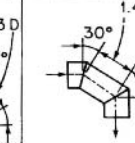
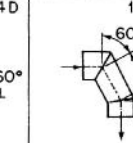
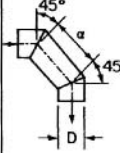
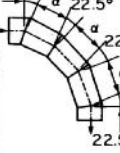
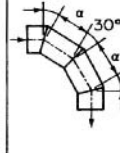
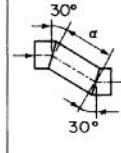


**FIGURE 42** Resistance coefficients for bends of uniform diameter and smooth surface at Reynolds number  $\approx 2.25 \times 10^5$  (Hydraulic Institute Engineering Data Book, Reference 5).

**PUMP SUCTION ELBOWS** Figures 43 and 44 illustrate two typical rectangular to round reducing suction elbows. Elbows of this configuration are sometimes used under dry-pit vertical volute pumps. These elbows are formed in concrete and are designed to require a minimum height, thus permitting a higher pump setting with reduced excavation. Figure 43 shows a long-radius elbow, and Figure 44 a short-radius elbow. The resulting velocity distribution into the impeller eye and the loss of head are shown for these two designs.

**METERS** Orifice, nozzle, and venturi meters (Figures 45–47) are used to measure rate of flow. These meters, however, introduce additional loss of head into the pumping system. Each of these meters is designed to create a pressure differential through the primary element. The magnitude of the pressure differential depends on the velocity and density of the liquid and the design of the element. The primary element restricts the area of flow, increases the velocity, and decreases the pressure. An expanding section following the pri-

**TABLE 7** Resistance coefficients for miter bends at reynolds number  $\approx 2.25 \times 10^5$

 <p><math>K_s = 0.016</math> <math>K_r = 0.024</math></p>	 <p><math>K_s = 0.034</math> <math>K_r = 0.044</math></p>	 <p><math>K_s = 0.042</math> <math>K_r = 0.062</math></p>	 <p><math>K_s = 0.065</math> <math>K_r = 0.154</math></p>	 <p><math>K_s = 0.130</math> <math>K_r = 0.165</math></p>	 <p><math>K_s = 0.236</math> <math>K_r = 0.320</math></p>	 <p><math>K_s = 0.471</math> <math>K_r = 0.684</math></p>	 <p><math>K_s = 1.129</math> <math>K_r = 1.265</math></p>																																																																																																																							
 <p><math>K_s = 0.112</math> <math>K_r = 0.204</math></p>	 <p><math>K_s = 0.150</math> <math>K_r = 0.268</math></p>	 <p><math>K_s = 0.143</math> <math>K_r = 0.227</math></p>	 <p><math>K_s = 0.108</math> <math>K_r = 0.236</math></p>	 <p><math>K_s = 0.188</math> <math>K_r = 0.320</math></p>	 <p><math>K_s = 0.202</math> <math>K_r = 0.323</math></p>	 <p><math>K_s = 0.400</math> <math>K_r = 0.534</math></p>	 <p><math>K_s = 0.400</math> <math>K_r = 0.601</math></p>																																																																																																																							
 <table><tr><th><math>a/D</math></th><th><math>K_s</math></th><th><math>K_r</math></th></tr><tr><td>0.71</td><td>0.507</td><td>0.510</td></tr><tr><td>0.943</td><td>0.230</td><td>0.415</td></tr><tr><td>1.174</td><td>0.333</td><td>0.384</td></tr><tr><td>1.42</td><td>0.261</td><td>0.377</td></tr><tr><td>1.50*</td><td>0.280</td><td>0.376</td></tr><tr><td>1.88</td><td>0.269</td><td>0.390</td></tr><tr><td>2.58</td><td>0.338</td><td>0.429</td></tr><tr><td>3.14</td><td>0.346</td><td>0.426</td></tr><tr><td>3.72</td><td>0.356</td><td>0.490</td></tr><tr><td>4.89</td><td>0.389</td><td>0.455</td></tr><tr><td>5.59</td><td>0.392</td><td>0.444</td></tr><tr><td>6.29</td><td>0.399</td><td>0.444</td></tr></table>	$a/D$	$K_s$	$K_r$	0.71	0.507	0.510	0.943	0.230	0.415	1.174	0.333	0.384	1.42	0.261	0.377	1.50*	0.280	0.376	1.88	0.269	0.390	2.58	0.338	0.429	3.14	0.346	0.426	3.72	0.356	0.490	4.89	0.389	0.455	5.59	0.392	0.444	6.29	0.399	0.444	 <table><tr><th><math>a/D</math></th><th><math>K_s</math></th><th><math>K_r</math></th></tr><tr><td>1.86</td><td>0.120</td><td>0.294</td></tr><tr><td>1.40</td><td>0.125</td><td>0.252</td></tr><tr><td>1.50*</td><td>—</td><td>0.250</td></tr><tr><td>2.25*</td><td>0.163</td><td>0.124</td></tr><tr><td>1.86</td><td>0.117</td><td>0.272</td></tr><tr><td>2.325</td><td>0.096</td><td>0.317</td></tr><tr><td>2.40*</td><td>0.095</td><td>—</td></tr><tr><td>2.91</td><td>0.108</td><td>0.317</td></tr><tr><td>3.49</td><td>0.130</td><td>0.318</td></tr><tr><td>4.65</td><td>0.148</td><td>0.310</td></tr><tr><td>6.05</td><td>0.142</td><td>0.313</td></tr></table>	$a/D$	$K_s$	$K_r$	1.86	0.120	0.294	1.40	0.125	0.252	1.50*	—	0.250	2.25*	0.163	0.124	1.86	0.117	0.272	2.325	0.096	0.317	2.40*	0.095	—	2.91	0.108	0.317	3.49	0.130	0.318	4.65	0.148	0.310	6.05	0.142	0.313	 <table><tr><th><math>a/D</math></th><th><math>K_s</math></th><th><math>K_r</math></th></tr><tr><td>1.23</td><td>0.195</td><td>0.347</td></tr><tr><td>1.44</td><td>0.196</td><td>0.320</td></tr><tr><td>1.67</td><td>0.150</td><td>0.300</td></tr><tr><td>1.70*</td><td>0.149</td><td>0.299</td></tr><tr><td>1.91</td><td>0.154</td><td>0.312</td></tr><tr><td>2.37</td><td>0.167</td><td>0.337</td></tr><tr><td>2.96</td><td>0.172</td><td>0.342</td></tr><tr><td>4.11</td><td>0.190</td><td>0.354</td></tr><tr><td>4.70</td><td>0.192</td><td>0.360</td></tr><tr><td>6.10</td><td>0.201</td><td>0.360</td></tr></table>	$a/D$	$K_s$	$K_r$	1.23	0.195	0.347	1.44	0.196	0.320	1.67	0.150	0.300	1.70*	0.149	0.299	1.91	0.154	0.312	2.37	0.167	0.337	2.96	0.172	0.342	4.11	0.190	0.354	4.70	0.192	0.360	6.10	0.201	0.360	 <table><tr><th><math>a/D</math></th><th><math>K_s</math></th><th><math>K_r</math></th></tr><tr><td>1.23</td><td>0.157</td><td>0.300</td></tr><tr><td>1.67</td><td>0.156</td><td>0.378</td></tr><tr><td>2.37</td><td>0.143</td><td>0.264</td></tr><tr><td>3.77</td><td>0.160</td><td>0.242</td></tr></table>	$a/D$	$K_s$	$K_r$	1.23	0.157	0.300	1.67	0.156	0.378	2.37	0.143	0.264	3.77	0.160	0.242
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1.67	0.150	0.300																																																																																																																												
1.70*	0.149	0.299																																																																																																																												
1.91	0.154	0.312																																																																																																																												
2.37	0.167	0.337																																																																																																																												
2.96	0.172	0.342																																																																																																																												
4.11	0.190	0.354																																																																																																																												
4.70	0.192	0.360																																																																																																																												
6.10	0.201	0.360																																																																																																																												
$a/D$	$K_s$	$K_r$																																																																																																																												
1.23	0.157	0.300																																																																																																																												
1.67	0.156	0.378																																																																																																																												
2.37	0.143	0.264																																																																																																																												
3.77	0.160	0.242																																																																																																																												

$K_s$  = RESISTANCE COEFFICIENT FOR SMOOTH SURFACE  
 $K_r$  = RESISTANCE COEFFICIENT FOR ROUGH SURFACE,  $\epsilon \approx 0.0022$

\*OPTIMUM VALUE OF  $\alpha$  INTERPOLATED

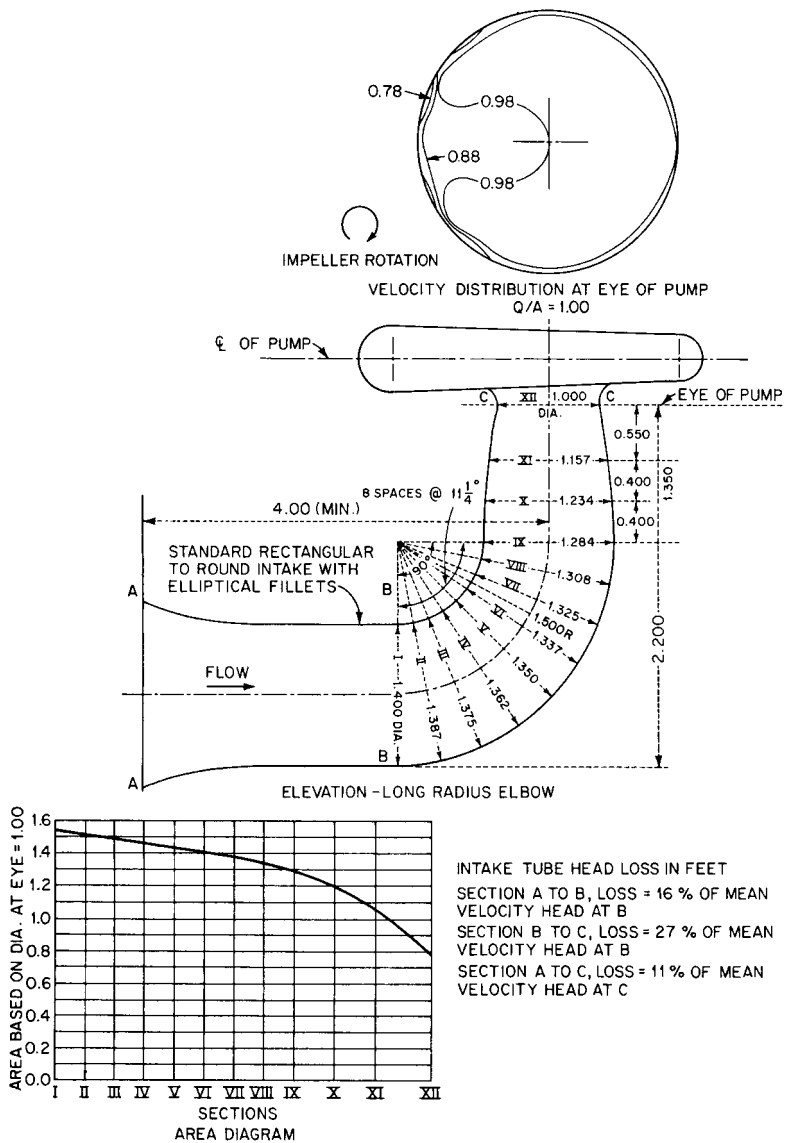
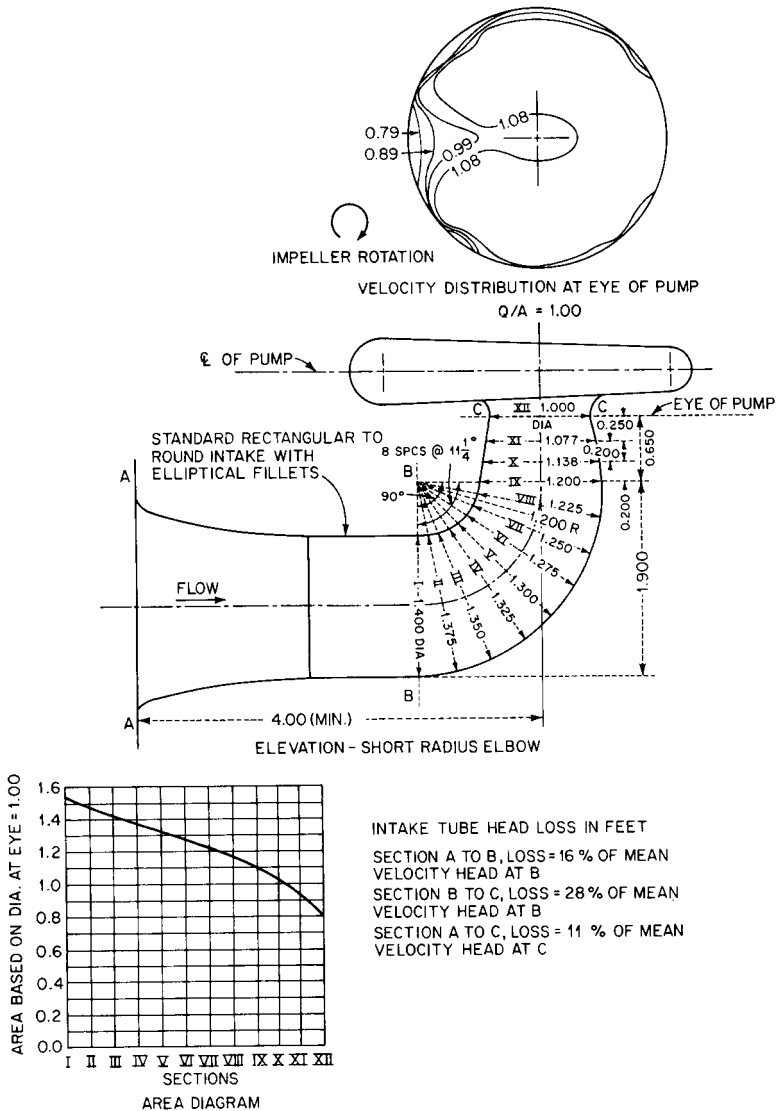


FIGURE 43 Head loss in a long-radius pump suction elbow (in  $\times 25.4 =$  mm) (Reference 16)

many element provides pressure head recovery and determines the meter efficiency. The pressure differential between inlet and throat taps measure rate of flow; the pressure differential between inlet and outlet taps measures the meter head loss (an outlet tap is not usually provided). Of the three types, venturi meters offer the least resistance to flow, and orifice meters the most.



**FIGURE 44** Head loss in a short-radius pump suction elbow (in  $\times 25.4 = \text{mm}$ ) (Reference 16)

When meters are designed and pressure taps located as recommended,<sup>6</sup> Figures 48–50 may be used to estimate the overall pressure loss. In these figures, the loss of pressure is expressed as a percentage of the differential pressure measured at the appropriate taps and values are given for various sizes of meters. This loss of pressure is also the meter total head, or energy loss, because there is no change in velocity head if the pipe inside diameters are the same at the various measuring points. The meter loss of head should be in units of feet (meters) of liquid pumped if other system losses are expressed this way.

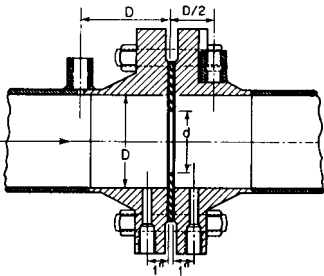


FIGURE 45 Thin-plate, square-edged orifice meter, showing alternate locations of pressure taps (Reference 6)

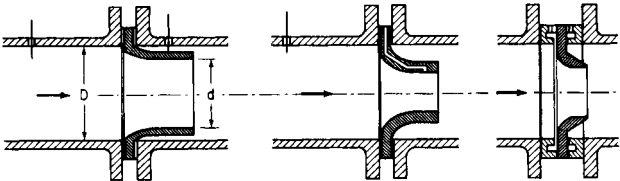


FIGURE 46 Shapes of flow nozzle meters and locations of pressure taps (Reference 6)

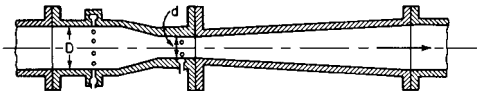


FIGURE 47 Herschel-type venturi meter, showing locations of pressure taps (Reference 6)

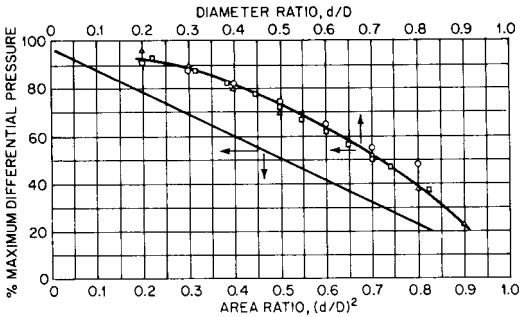
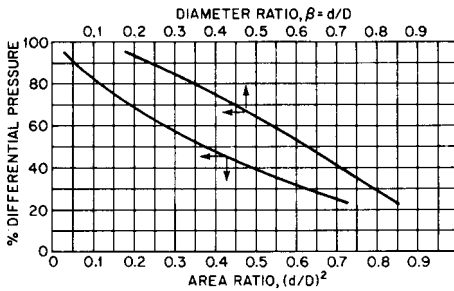


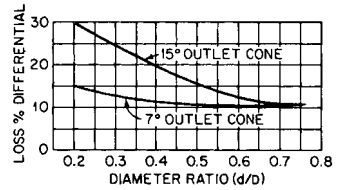
FIGURE 48 Overall pressure loss across thin-plate orifices (Reference 6)

Reference 6 should be consulted for information concerning formulas and coefficients for calculating differential pressure versus rate of flow.

**Screens, Perforated Plates, and Bar Racks** Obstructions to the flow of liquid in the form of multiple orifices uniformly distributed across an open or closed conduit may be used to remove solids, throttle flow, and produce or reduce turbulence. They may be used



**FIGURE 49** Overall pressure loss across flow nozzles (Reference 6)



**FIGURE 50** Overall pressure loss across venturi tubes (Reference 6)

upstream or downstream from a pump, depending on their purpose, and they therefore introduce a loss of head that must be accounted for. When an obstruction is placed upstream from a pump, a significant reduction in suction pressure and *NPSH* available can result.

The loss of head results from an increase in velocity at the entrance to the openings, friction, and the sudden decrease in velocity following the expansion of the numerous liquid jets. The total head loss is a function of the ratio of the total area of the openings to the area of the conduit before the obstruction, the thickness of the obstruction, the Reynolds number, and the velocities. Various investigators have determined values for resistance coefficients that can be multiplied by the approach velocity head to obtain the loss through these obstructions. According to Idel'chik,<sup>7</sup> loss of head  $h$  in feet (meters) may be calculated from the equation

$$h = K_1 \frac{V_1^2}{2g} \quad (24)$$

where  $K_1$  = resistance coefficient

$V_1$  = average velocity in the conduit approaching the obstruction, ft/s (m/s)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

**ROUND-WIRE MESH SCREENS** For flow having Reynolds numbers equal to or greater than 400, the resistance coefficient for flow through a round-wire, plain square mesh screen (Figure 51a) may be estimated as a function of percentage of open area using the equations

$$\text{in USCS units} \quad Re = \frac{V_o W_d}{\nu 12} \geq 400 \quad (25a)$$

$$\text{in SI units} \quad Re = \frac{V_o W_d}{\nu 1000} \geq 400 \quad (25b)$$

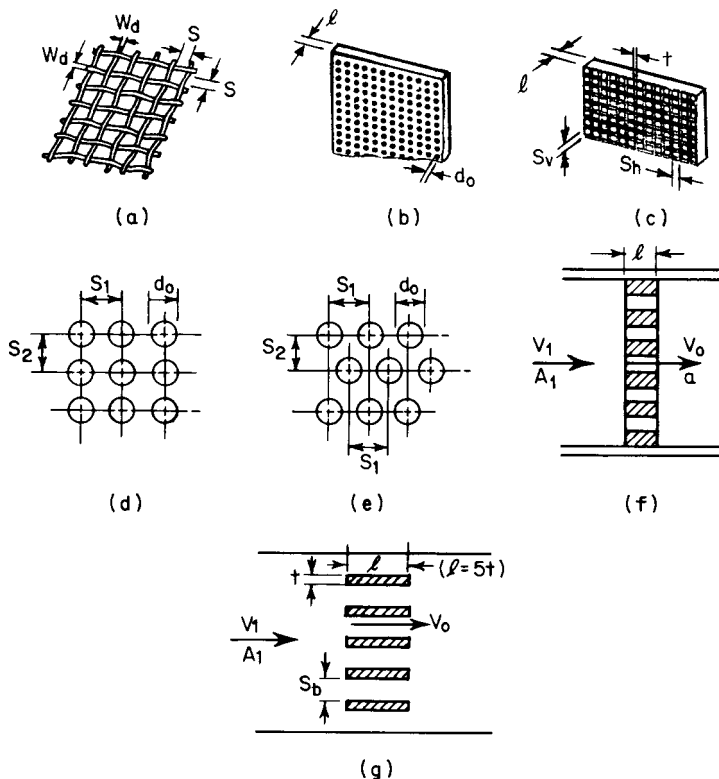
$$K_1 = k_0 \left( 1 - \frac{A_r}{100} \right) + \left( \frac{100}{A_r} - 1 \right)^2 \quad (26)$$

$$A_r = 100 \left( \frac{S}{S + W_d} \right)^2 \quad (27)$$

$$A_r = 100(1 - MW_d)^2 \quad (28)$$

where  $Re$  = screen Reynolds number referred to wire diameter

$V_o$  = velocity through area of rectangular opening =  $100V_1/A_r$ , ft/s (m/s)



**FIGURE 51** Explanation of terms used in Eq. 25–28 and 30–37 and Tables 8 and 9 for calculating resistance coefficient  $K_1$  (a) round-wire, plain square, mesh screen, (b) round-hole perforated plate, (c) rectangular-grid perforated plate, (d) perforated plate, holes in vertical columns, (e) perforated plate, staggered holes, (f) perforated plate, cross section, (g) rectangular bar rack, cross section

$W_d$  = screen wire diameter, in (mm)

$\nu$  = kinematic viscosity,  $\text{ft}^2/\text{s}$  ( $\text{m}^2/\text{s}$ )

$k_0$  = 1.0 for new, perfectly clean screens to 1.3 for normal screens

$A_r$  = percentage of open area

$S$  = square space between wires, in (mm)

$M$  = mesh of screen or number of wires per in (mm)

Using water in a flow range of Reynolds numbers of approximately 60 to 800, Padmanabhan and Vigander<sup>8</sup> experimented with 1.3- to 12.0-mesh screens, 51.4 to 56% open area, and found their results to be comparable with those of other investigators. Values of the coefficient of resistance  $K_1$ , from the Padmanabhan-Vigander tests and others, vary from 2 decreasing asymptotically to 1 with increase in Reynolds number (Eq. 25) up to approximately 1000 for 47 to 56% open area. Smaller percentage open area values have larger coefficients.

Armour and Cannon,<sup>9</sup> using nitrogen as the fluid, tested various types of fine woven wire screens and derived equations for pressure drop in terms of a friction factor and the screen Reynolds number. Approach velocities ranged from 0.1 to 30  $\text{ft/s}$  (0.03 to 9  $\text{m/s}$ ),

**TABLE 8** Plain square screen geometry

Screen mesh size $M$ , in <sup>-1</sup> (mm <sup>-1</sup> )	Wire diameter $W_d$ , in $\times 10^{-4}$ (mm $\times 10^{-4}$ )	Open area $A_r$ , %	Screen constant $C_1$ , ft <sup>-1</sup> $\times 10^3$ (m <sup>-1</sup> $\times 10^3$ )	Screen constant $C_2$
30 $\times$ 30 (1.18 $\times$ 1.18)	9.45 (240)	94.4	2.553 (8.376)	1.553
150 $\times$ 150 (5.91 $\times$ 5.91)	2.36 (59.9)	93.0	8.267 (27.12)	2.594
250 $\times$ 250 (9.84 $\times$ 9.84)	1.69 (42.9)	91.7	16.21 (53.18)	2.934
400 $\times$ 400 (15.7 $\times$ 15.7)	1.00 (25.4)	92.2	30.68 (100.7)	2.680

Source: Reference 9.

resulting in Reynolds numbers from 350 to 275,000. To simplify calculations, screen geometry constants  $C_1$  = (surface area to unit volume ratio)<sup>2</sup>  $\times$  (pore diameter) and  $C_2$  = (screen thickness)  $\div$  (void fraction)<sup>2</sup>  $\times$  (pore diameter) have been added to the reference authors' equations to obtain the following expression for screen head loss  $h$  in feet (meters):

$$h = C_2 \frac{V_1}{g} (8.61vC_1 + 0.52V_1) \quad (29)$$

where  $V_1$  = average velocity in the conduit approaching the screen, ft/s (m/s)

Table 8 lists  $C_1$  and  $C_2$  values for a sample of plain square screens tested by Armour and Cannon.

**PERFORATED PLATES AND BAR RACKS** For flow having Reynolds numbers equal to or greater than  $10^5$ , the resistance coefficients for flow through thick, square-edge perforated plates with round (Figure 51b) or rectangular (Figure 51c) openings and racks with rectangular cross-section bars (length = 5 times thickness; Figure 51g) may be calculated using Eq. 24, Table 9, and the following equations:

$$Re = \frac{V_o D_h}{\nu} \geq 10^5 \quad (30)$$

$$\text{In USCS units} \quad D_h = \frac{a}{3p} \quad \text{or} \quad D_h = \frac{d_o}{12} \quad (31a)$$

$$\text{In SI units} \quad D_h = 0.004 \frac{a}{\rho} \quad \text{or} \quad D_h = \frac{d_o}{1000} \quad (31b)$$

$$\text{for plates with round holes} \quad A_r = 100 \left( \frac{0.785 d_o^2}{S_1 S_2} \right) \quad (32)$$

$$\text{for plates with single hole in center} \quad A_r = 100 \left( \frac{d_o}{d_1} \right)^2 \quad (33)$$

$$\text{for plates with square openings } (S = S_h = S_v) \quad A_r = 100 \left( \frac{S}{S + t} \right)^2 \quad (34)$$

$$\text{for plates with rectangular openings} \quad A_r = 100 \left[ \frac{S_h S_v}{(S_h + t)(S_v + t)} \right] \quad (35)$$

$$\text{for any plate or bar rack} \quad A_r = 100 \frac{A_o}{A_1} \quad (36)$$

$$\text{for single vertical or horizontal bar racks} \quad A_r = 100 \left( \frac{S_b}{S_b + t} \right) \quad (37)$$



**TABLE 9** Values of resistance coefficient  $K_1$  for perforated plates and bar racks

$l/Dh$	$A_r/100$ or $A_o/A_1$															
	0.02	0.04	0.06	0.08	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0
0	7000	1670	730	400	245	96.0	51.5	30.0	18.2	8.25	4.00	2.00	0.97	0.42	0.13	0
0.2	6600	1600	687	374	230	94.0	48.0	28.0	17.4	7.70	3.75	1.87	0.91	0.40	0.13	0.01
0.4	6310	1530	660	356	221	89.0	46.0	26.5	16.6	7.40	3.60	1.80	0.88	0.39	0.13	0.01
0.6	5700	1380	590	322	199	81.0	42.0	24.0	15.0	6.60	3.20	1.60	0.80	0.36	0.13	0.01
0.8	4680	1130	486	264	164	66.0	34.0	19.6	12.2	5.50	2.70	1.34	0.66	0.31	0.12	0.02
1.0	4260	1030	443	240	149	60.0	31.0	17.8	11.1	5.00	2.40	1.20	0.61	0.29	0.11	0.02
1.4	3930	950	408	221	137	55.6	28.4	16.4	10.3	4.60	2.25	1.15	0.58	0.28	0.11	0.03
2.0	3770	910	391	212	134	53.0	27.4	15.8	9.90	4.40	2.20	1.13	0.58	0.28	0.12	0.04
3.0	3765	913	392	214	132	53.5	27.5	15.9	10.0	4.50	2.24	1.17	0.61	0.31	0.15	0.06
4.0	3775	930	400	215	132	53.8	27.7	16.2	10.0	4.60	2.25	1.20	0.64	0.35	0.16	0.08
5.0	3850	936	400	220	133	55.5	28.5	16.5	10.5	4.75	2.40	1.28	0.69	0.37	0.19	0.10
6.0	3870	940	400	222	133	55.8	28.5	16.6	10.5	4.80	2.42	1.32	0.70	0.40	0.21	0.12
7.0	4000	950	405	230	135	55.9	29.0	17.0	10.9	5.00	2.50	1.38	0.74	0.43	0.23	0.14
8.0	4000	965	410	236	137	56.0	30.0	17.2	11.1	5.10	2.58	1.45	0.80	0.45	0.25	0.16
9.0	4080	985	420	240	140	57.0	30.0	17.4	11.4	5.30	2.62	1.50	0.82	0.50	0.28	0.18
10	4110	1000	430	245	146	59.7	31.0	18.2	11.5	5.40	2.80	1.57	0.89	0.53	0.32	0.20

Note:  $l$  = thickness of perforated plate or length of bars, ft (m);  $D_h$ ,  $A_r$ ,  $A_o$ , and  $A_1$  are as defined following Eq. 37.

Source: Reference 7.

where  $Re$  = Reynolds number referred to hydraulic diameter

$V_o$  = velocity through area of opening, ft/s (m/s)

$D_h$  = hydraulic diameter (= diameter if openings are round holes), ft (m)

$\nu$  = kinematic viscosity, ft<sup>2</sup>/s (m<sup>2</sup>/s)

$a$  = area of single opening, in<sup>2</sup> (mm<sup>2</sup>)

$p$  = perimeter of single opening, in (mm)

$d_o$  = diameter of hole, in (mm)

$A_r$  = percentage of open area

$S_1$  = horizontal spacing of holes, in (mm)

$S_2$  = vertical spacing of holes, in (mm)

$d_1$  = diameter of approach, in (mm)

$S_h$  = horizontal apace between vertical bars, in (mm)

$S_v$  = vertical space between horizontal bars, in (mm)

$t$  = thickness of plate laths or bars, in (mm)

$A_o$  = total area of openings, ft<sup>2</sup> (m<sup>2</sup>)

$A_1$  = total area of approach, ft<sup>2</sup> (m<sup>2</sup>)

$S_b$  = space between single vertical or horizontal bars, in (mm)

The loss of head through perforated plates may also be calculated by using an *orifice coefficient*  $C$ , as suggested by Smith and Van Winkle<sup>10</sup> and Kolodzie and Van Winkle.<sup>11</sup> Test results using air and other gases with equilateral-triangle pitch perforated plates are shown in Table 10 for Reynolds numbers 400 to 20,000. Using single-orifice relations, the following expression equates flow rate to pressure drop:

$$\omega = CA_o \sqrt{\frac{2g\Delta P}{\gamma \left[ 1 - \left( \frac{A_r}{100} \right)^2 \right]}} \quad (38)$$

where  $\omega$  = flow rate, ft<sup>3</sup>/s (m<sup>3</sup>/s)

$C$  = orifice coefficient from Table 10

$A_o$  = total area of openings, ft<sup>2</sup> (m<sup>2</sup>)

$g$  = acceleration of gravity, 32.17 ft/s<sup>2</sup> (9.807 m/s<sup>2</sup>)

$\Delta P$  = pressure drop, lb/ft<sup>2</sup> (N/m<sup>2</sup>)

$\gamma$  = specific weight (force) of liquid, lb/ft<sup>3</sup> (N/m<sup>3</sup>)

$A_r$  = percentage of open area

The resistance coefficient  $K_1$  and the orifice coefficient  $C$  may be interchanged in Eqs. 24 and 38 for loss through perforated plates:

$$C = \sqrt{\frac{1 - \left( \frac{A_r}{100} \right)^2}{K_1 - \left( \frac{A_r}{100} \right)^2}} \quad (39)$$

$$K_1 = \frac{1 - \left( \frac{A_r}{100} \right)^2}{C^2 \left( \frac{A_r}{100} \right)^2} \quad (40)$$

**TABLE 10** Orifice coefficients  $C$  for equilateral-triangle pitch perforated plates

Pitch/ $D_h$	$Re$	$l/D_h$								
		0.33	0.43	0.50	0.52	0.65	0.75	0.80	1.0	2.0
2.0	400–4000	0.74–0.69	0.75–0.71	0.76–0.74	—	0.79–0.85	0.82–0.86	0.83–0.87	0.84–0.89	0.77–0.92
2.0	4000–20,000	0.69	0.71	0.74	0.77	0.85	0.86	0.87	0.89	0.92
3.0	400–4000	0.70–0.67	0.72–0.68	0.73–0.72	—	0.75–0.81	0.76–0.82	0.77–0.84	0.78–0.85	0.70–0.87
3.0	4000–20,000	0.67	0.68	0.72	0.74	0.81	0.82	0.84	0.85	0.87
4.0	400–4000	0.71–0.66	0.72–0.67	0.72–0.69	—	0.73–0.77	0.74–0.79	0.75–0.81	0.76–0.83	0.69–0.84
4.0	4000–20,000	0.66	0.67	0.69	0.72	0.77	0.79	0.81	0.88	0.84
5.0	400–4000	0.68–0.65	0.69–0.66	0.7–0.68	—	0.72–0.76	0.73–0.77	0.74–0.78	0.75–0.81	0.65–0.82
5.0	4000–20,000	0.65	0.66	0.68	0.71	0.76	0.77	0.78	0.81	0.82

Note: Pitch/ $D_h$  = pitch-to-hole-diameter ratio;  $l/D_h$  = plate-thickness-to-hole diameter ratio;  $Re$  = Reynolds number

Source: References 10 and 11.

**Throttling Orifices** In addition to measuring flow, orifices can be used to (a) reduce flow by adding artificial resistance to increase system head, (b) dissipate energy to provide a desired pressure reduction, and (c) create a high-velocity jet. Orifices for these purposes are called *throttling orifices*. In order to maintain a desired minimum flow to prevent damage to a centrifugal pump, a throttling orifice, or a series of throttling orifices, can be used in the bypass system. The orifice provides additional bypass resistance to maintain the required bypass flow.

A throttling orifice can be fabricated by drilling a hole in a metal plate (or through bar stock) that, when inserted between flanges in a pipe (or threaded to pipe), will create the desired loss of head at the design flow. The resistance coefficient  $K_1$  may be calculated as if the throttling device were a single hole in a perforated plate, using Table 9 and Eqs. 30, 31, and 33, and the loss of head calculated using Eq. 24.

Energy is dissipated through a throttling orifice because pressure head is converted to velocity head and this conversion is followed by an inefficient pressure head recovery. Conditions may exist at the orifice *vena contracta* that could cause vaporization of the high-velocity, low-pressure liquid jet. Care must be taken in selecting the orifice size to avoid excessive cavitation noise or choke flow. An orifice cavitation index used to check the orifice selection is described by Tung and Mikasinovic,<sup>12</sup> who also discuss the use of orifices in series to avoid cavitation.

Although orifices are used to meter flow, accuracy requires that they be fabricated to standard proportions and that pressure taps be precisely located (Reference 6 and Figure 45). A distinction should be made between *meter differential pressure*, which is used to measure flow and is the difference in pressure at the upstream and downstream *vena contracta* taps, and *meter loss of head* calculated using the resistance coefficient  $K_1$ , which is the total overall loss of energy as measured at the upstream tap and past the downstream *vena contracta* tap. The loss of head through a standard orifice meter should be calculated as discussed previously under Meters.

**Water Meters and Backflow Preventors** Tables 11 and 12 from Reference 17 provide indicative values of losses through these devices.

## PUMP FLOW, HEAD, AND POWER IN VARYING TEMPERATURE SYSTEMS

In a pumping system where the weight of liquid pumped is constant, the volumetric flow rate will vary through system components having different temperatures. An example would be the condensate and feedwater system in a steam power plant. The following equation may be used to calculate volumetric flow rate using the specific gravity corresponding to the temperature of the liquid at the location in the system where flow is required:

$$\text{In USCS units} \quad \text{gpm} = \frac{\text{lb/h}}{500 \text{ (sp. gr.)}} \quad (41a)$$

$$\text{In SI units} \quad \text{m}^3/\text{h} = \frac{\text{kg/h}}{998 \text{ (sp. gr.)}} \quad (41b)$$

When calculating the total head required of a pump or pumps to overcome total system component losses, the actual volumetric flow rate and temperature through each component must be used because head loss is a function of velocity and viscosity. Information provided in this chapter permits computing pipe, valve, and fitting losses in  $\text{ft} \cdot \text{lb/lb}$  or  $\text{ft}$  ( $\text{N} \cdot \text{m/N}$  or  $\text{m}$ ) of liquid passing through the component. If the pump is at a location in the system where the temperature is different than at the locations where the head losses are calculated, the total head to be produced by the pump cannot be found by simply adding the individual component heads.

The pump head required to produce a specific increase in pressure varies inversely with specific gravity. Therefore, to calculate pump total head, either the component total

**TABLE 11** Pressure losses for turbine-type water meters

Flow Rate, gpm	Pressure Loss, psi, when Meter Size Is:						
	2 in	3 in	4 in	6 in	8 in	10 in	12 in
100	0.8	0.8	0.5				
150	3.5	3.5	1.5				
200	6.5	6.5	3.0				
250	10.0	10.0	4.5				
300		14.0	5.5				
350			7.0				
400			8.0	0.5			
450			9.0	1.0			
500			10.5	1.2	0.5	0.5	
600				2.0	1.0	0.6	
700				3.0	1.1	0.7	
800				4.0	1.4	0.8	
900				5.0	1.7	1.0	
1000				6.0	2.0	1.1	0.6
1500				13.0	4.5	2.0	1.0
2000					8.0	3.0	1.6
2500					13.0	4.0	3.0
3000						6.0	4.5
3500						8.5	6.0
4000						11.0	7.7
5000							11.0
6000							18.0

$\text{in} \times 25.4 \text{ mm} \qquad \text{gpm} \times .227 = \text{m}^3/\text{h} \qquad \text{psi} \times .069 = \text{bar}$

SOURCE: Reference 17

heads must be converted to pump equivalent heads and added together, or all component losses must be expressed in pressure units so the total of these pressures can be converted to an equivalent pump head at the pump temperature. From Eqs. 2 and 3, the pump equivalent head is

$$h_2 = \frac{\gamma_1}{\gamma_2} h_1$$

or

$$h_2 = \frac{\text{sp. gr.}_1}{\text{sp. gr.}_2} h_1$$

where the subscript 1 denotes the component and the subscript 2 denotes the pump. From Eq. 6, individual component head losses can be converted to lb/ft<sup>2</sup> (N/m<sup>2</sup>) using

$$p_1 = h_1 \gamma_1$$

Table 12 Pressure losses for backflow preventers

Flow Rate, gpm	Pressure Loss, psi, when Preventer Size Is:					
	2 in	3 in	4 in	6 in	8 in	10 in
100	3.0					
150	5.5	1.5				
200		2.2	3.0			
300		4.0	2.0			
400			2.5	3.5		
500			3.0	2.5		
600			4.0	2.4	4.5	
700			5.0	2.6	3.6	
800				2.8	3.0	
900				3.0	2.8	
1000				3.3	2.5	4.0
1200				3.7	2.0	3.5
1400				4.3	2.0	2.9
1600				5.0	2.4	2.7
1800					2.6	2.6
2000					2.8	2.5
2500					3.5	2.5
3000						3.0
3500						3.6
4000						4.0
4500						5.0

in  $\times$  25.4 mm      gpm  $\times$  .227 = m<sup>3</sup>/h      psi  $\times$  .069 = bar

SOURCE: Reference 17

Also for Eq. 6,

$$TH = \frac{P_{\Delta}}{\gamma_2} = \frac{\Sigma p_1}{\gamma_2}$$

from which total component losses in lb/ft<sup>2</sup> (N/m<sup>2</sup>) can be converted to an equivalent total pump head in feet (meters) of liquid to be produced at the pumping temperature.

In a varying temperature system, the positive or negative static head required to raise or lower the liquid pumped is not simply a difference in elevation. A pump must produce pressure in a pipe to raise liquid; the pressure required is proportional to the specific weight (force) of the liquid. The static head required at the pump should be found by expressing the suction and discharge elevation heads  $Z$  as pressures at the pump suction and discharge connections (corrected to the reference datum plane, if it is not at the pump centerline elevation) and using actual specific weights (forces) along the pipe. This differential pressure, in lb/ft<sup>2</sup> (N/m<sup>2</sup>), is then converted to an equivalent static head using the specific weight (force) or specific gravity at the pump in the above appropriate equations.

When designing a pumping system, there may be several locations for placing a pump to produce a specified flow rate in lb/h (kg/h) and an increase in pressure in lb/ft<sup>2</sup> (N/m<sup>2</sup>). If the temperature of the liquid varies at the different pump locations being considered (for example, before or after a feedwater heater in a steam power plant), the pump total head in feet (meters) and the volumetric flow rate in gpm (m<sup>3</sup>/h) will vary. Although it is true

that pump power is proportional to the product of volumetric flow  $\times$  head  $\times$  specific gravity, higher pumping temperature (lower specific gravity) will nevertheless result in higher pumping power. For the same conditions of weight (or mass) flow and differential pressure, pump power varies inversely with specific gravity because of the following relationships:

$$\text{Pump power} \propto \text{volumetric flow} \times \text{total head} \times \text{sp. gr.}$$

$$\text{Volumetric flow} \propto \frac{\text{weight or mass flow}}{\text{sp. gr.}}$$

$$\text{Total head} \propto \frac{\text{pressure}}{\text{sp. gr.}}$$

therefore,

$$\text{Pump power} \propto \frac{\text{weight or mass flow}}{\text{sp. gr.}} \times \frac{\text{pressure}}{\text{sp. gr.}} \times \text{sp. gr.}$$

then,

$$\text{Pump power} \propto \frac{(\text{weight or mass flow}) \times \text{pressure}}{\text{sp. gr.}}$$

Following are formulas for calculating pump input power in brake horsepower or brake kilowatts:

$$\text{In USCS units} \quad \text{bhp} = \frac{\text{gpm} \times TH \times \text{sp. gr.}}{3960 \times \text{pump eff.}} \quad (42a)$$

$$= \frac{\text{lb/h} \times \text{lb/in}^2}{858,600 \times \text{pump eff.} \times \text{sp. gr.}} \quad (43a)$$

$$\text{In SI units} \quad \text{bkW} = \frac{\text{m}^3/\text{h} \times \text{m} \times \text{sp. gr.}}{367.7 \times \text{pump eff.}} \quad (42b)$$

$$= \frac{\text{kg/h} \times \text{kPa}}{3,593,000 \times \text{pump eff.} \times \text{sp. gr.}} \quad (43b)$$

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